

IMN-359

Transformée de Fourier 2D

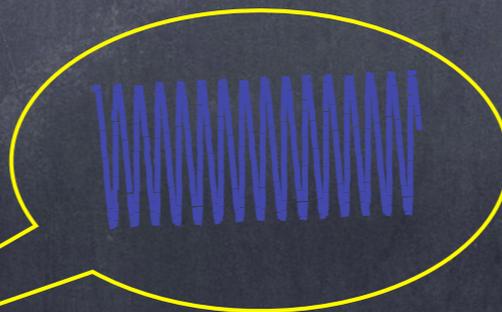
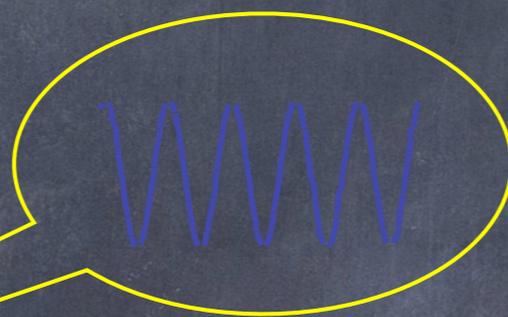
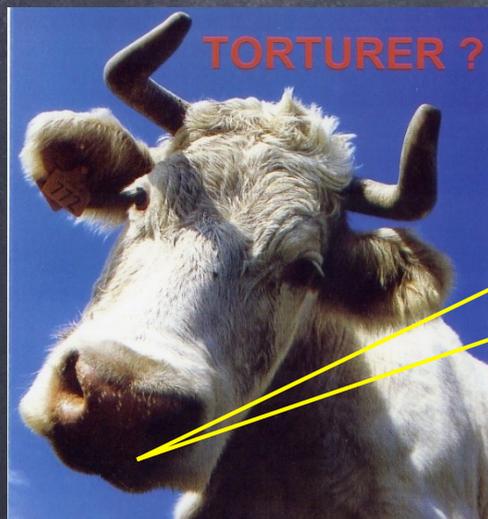
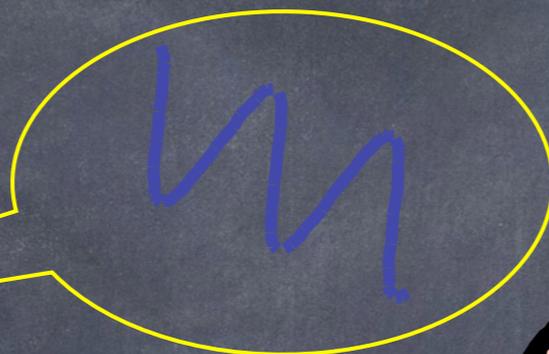
T.F discrète

- La transformée de Fourier

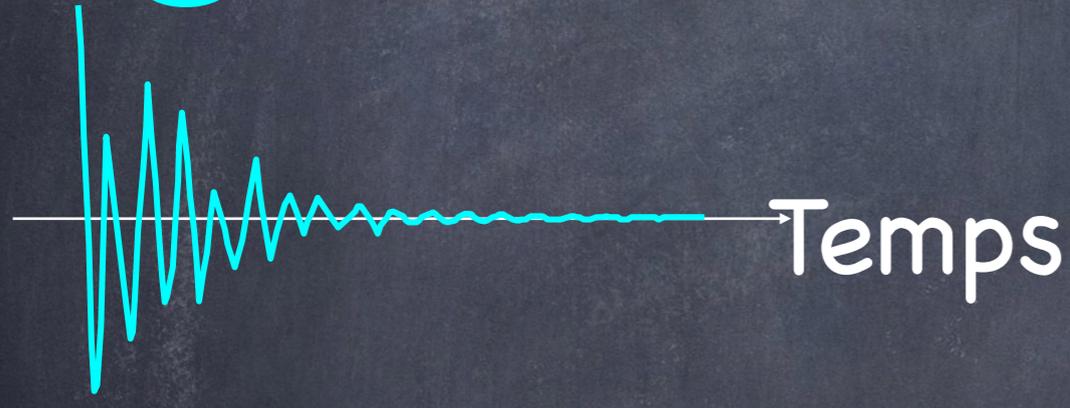
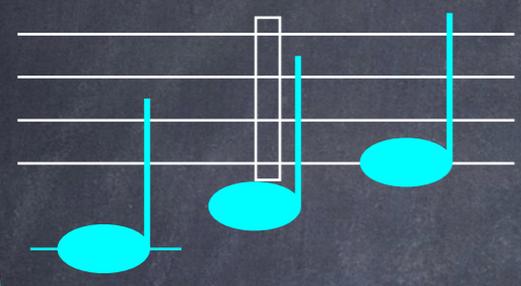


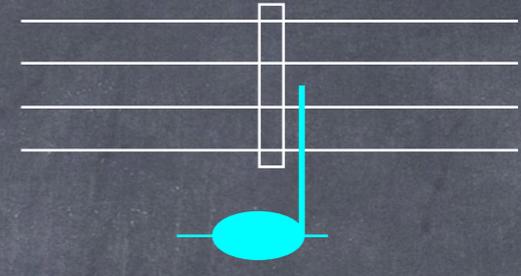
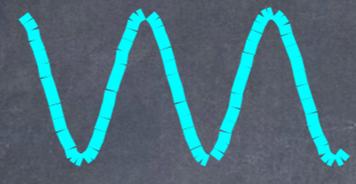
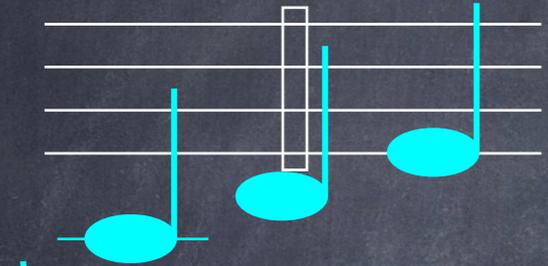
Notion de transformée de Fourier (TF)

Analogie : son = vibration qui se propage dans le temps

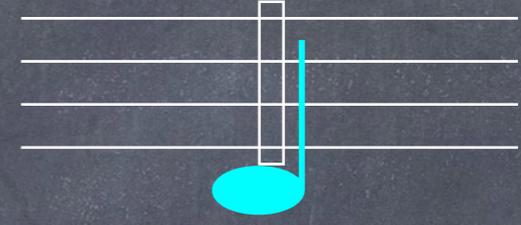


On entend non pas une vibration (fonction du temps) mais une note donc une fréquence

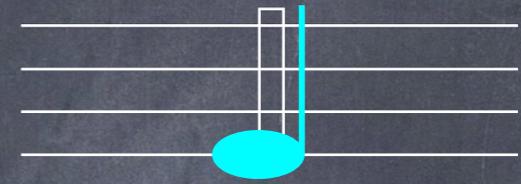




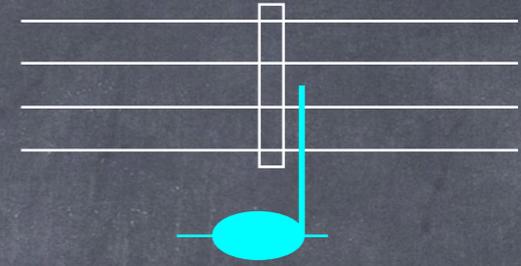
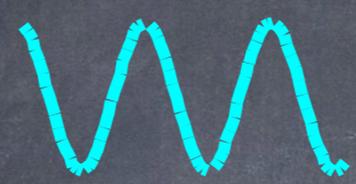
DO



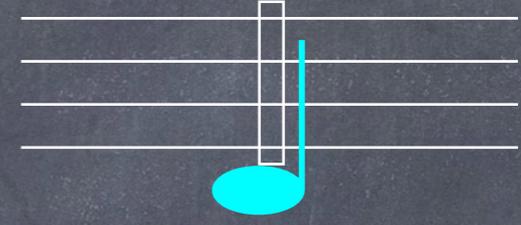
MI



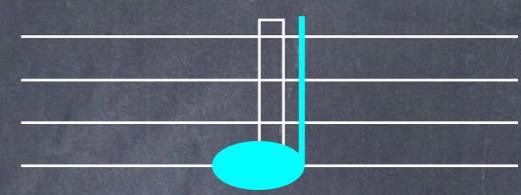
SOL



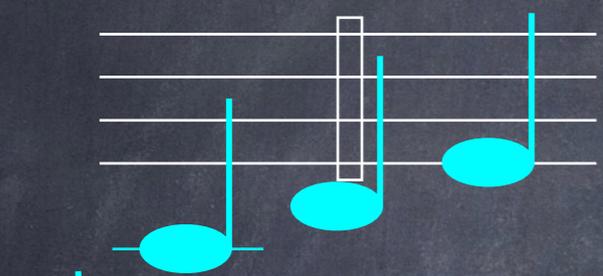
DO



MI



SOL



Intensité



DO MI SOL

Transformée de Fourier : donner le « poids » relatif d'une fréquence dans un signal

TF

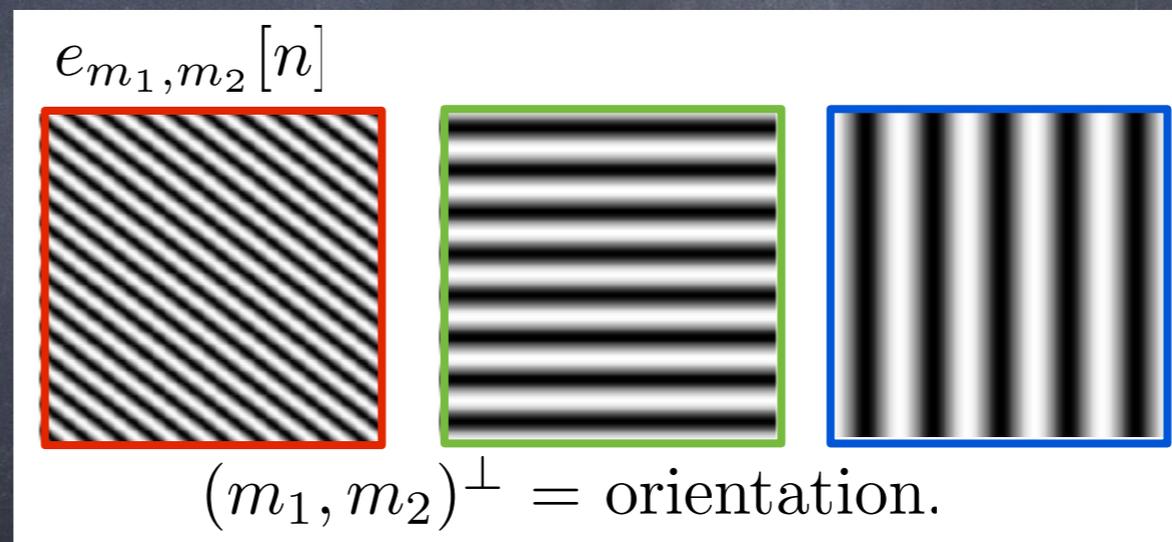
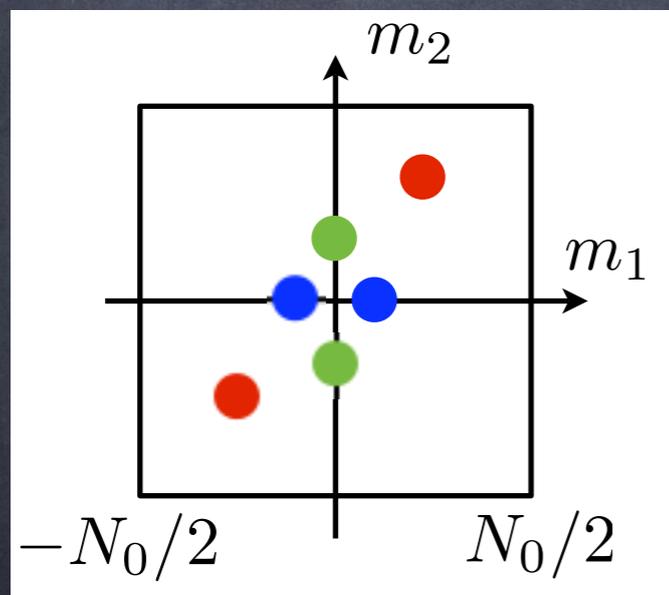


2D discrete Fourier Basis

2D discrete Fourier basis: $N = N_0 \times N_0$ pixels

$$e_m[n] = \frac{1}{\sqrt{N}} e^{\frac{2i\pi}{N_0} m_1 n_1 + \frac{2i\pi}{N_0} m_2 n_2} = e_{m_1}[n_1] e_{m_2}[n_2]$$

Frequency $m = (m_1, m_2) \in \{0, \dots, N_0 - 1\} \times \{0, \dots, N_0 - 1\}$



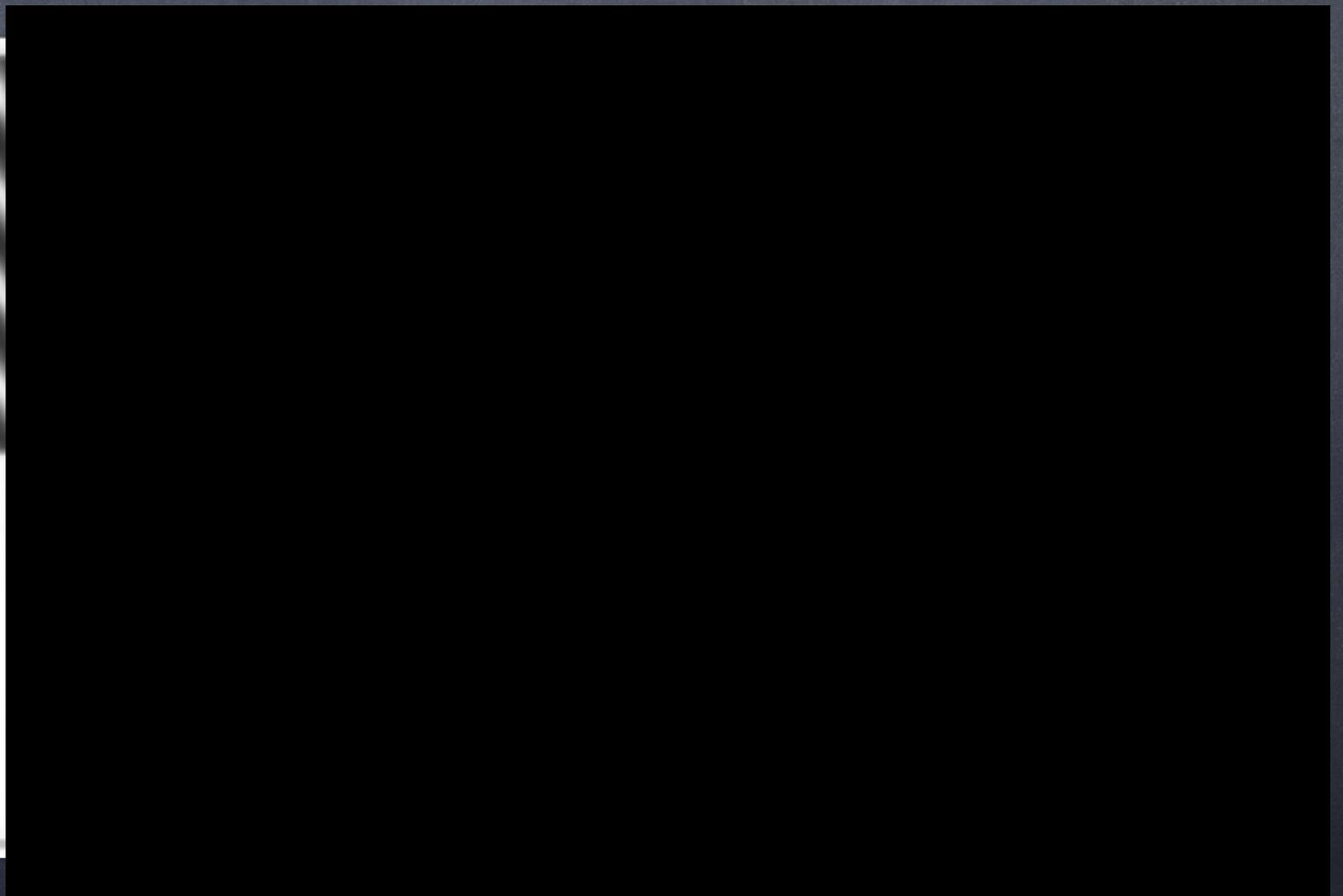
Et l'image, elle?

- ① Une image peut elle aussi être vue comme une somme d'ondes qui oscillent à différentes fréquences

Image



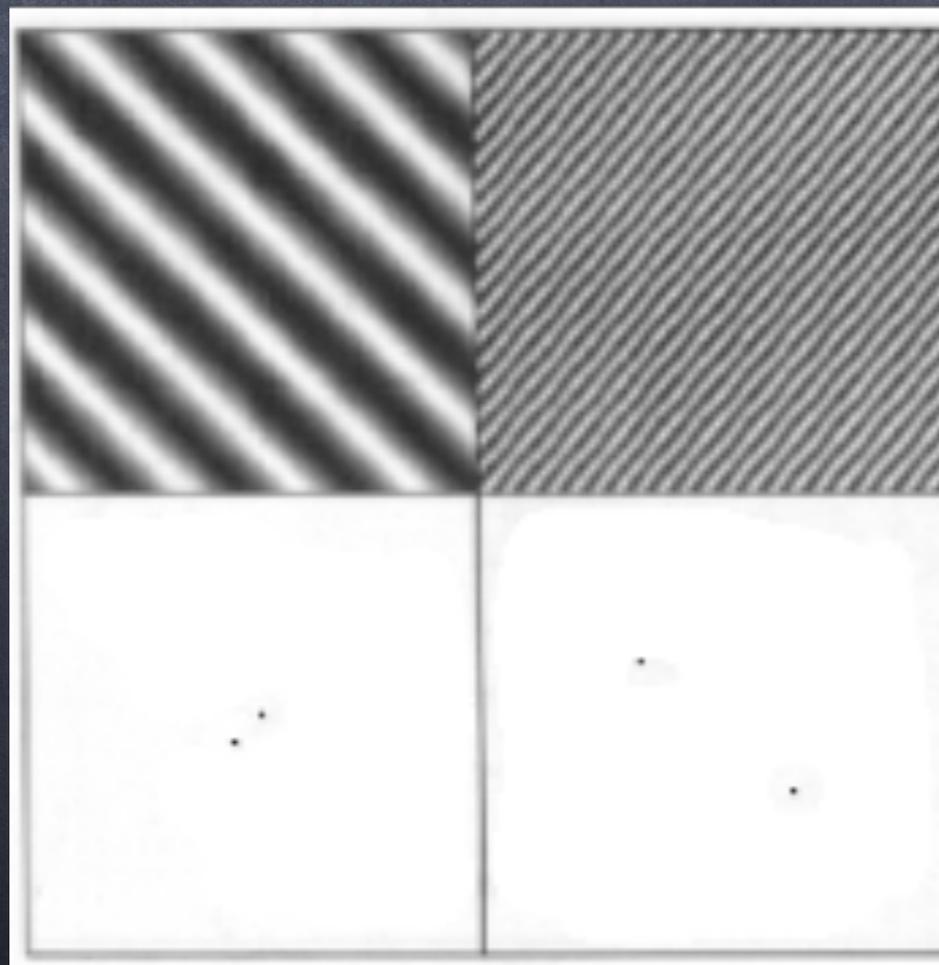
T.F.



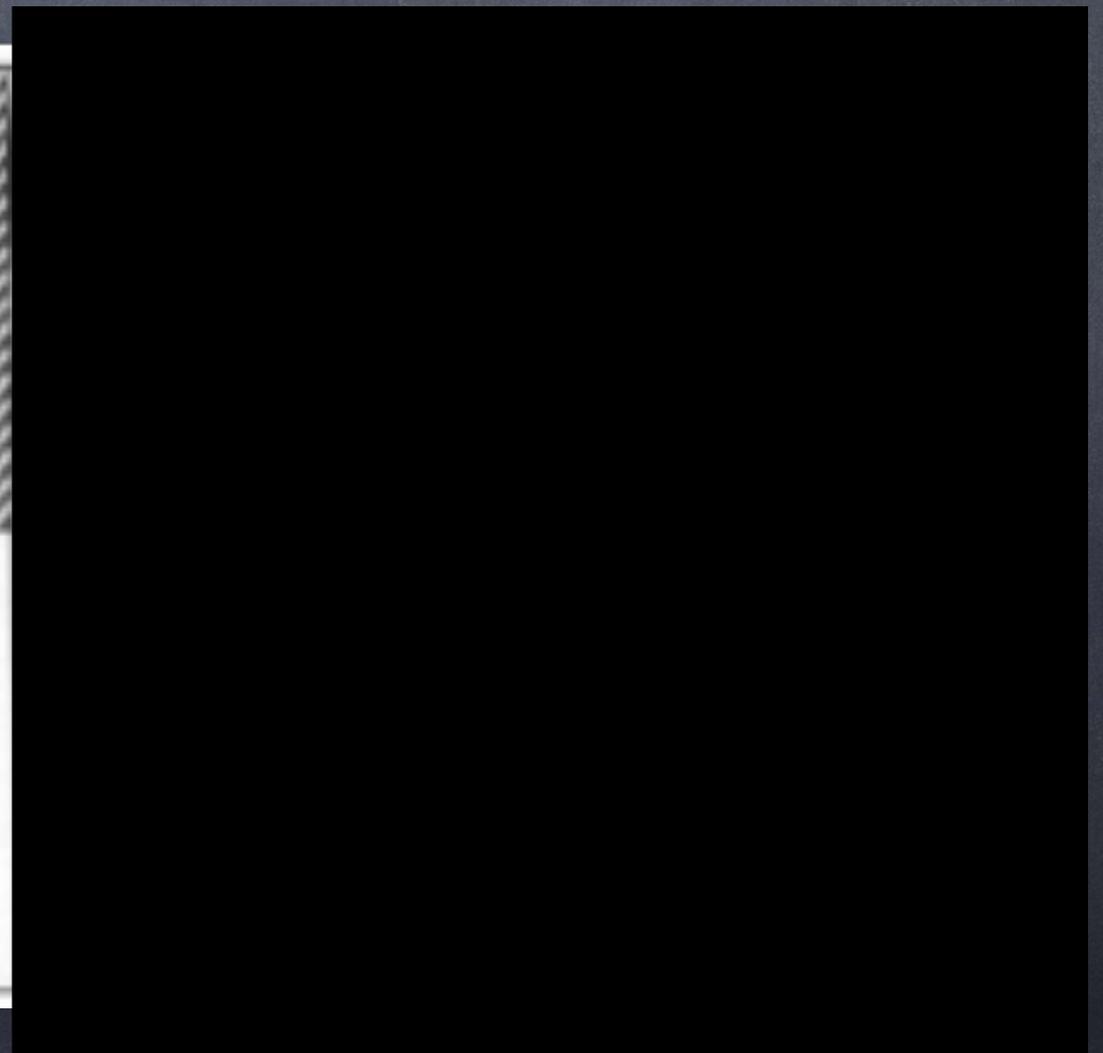
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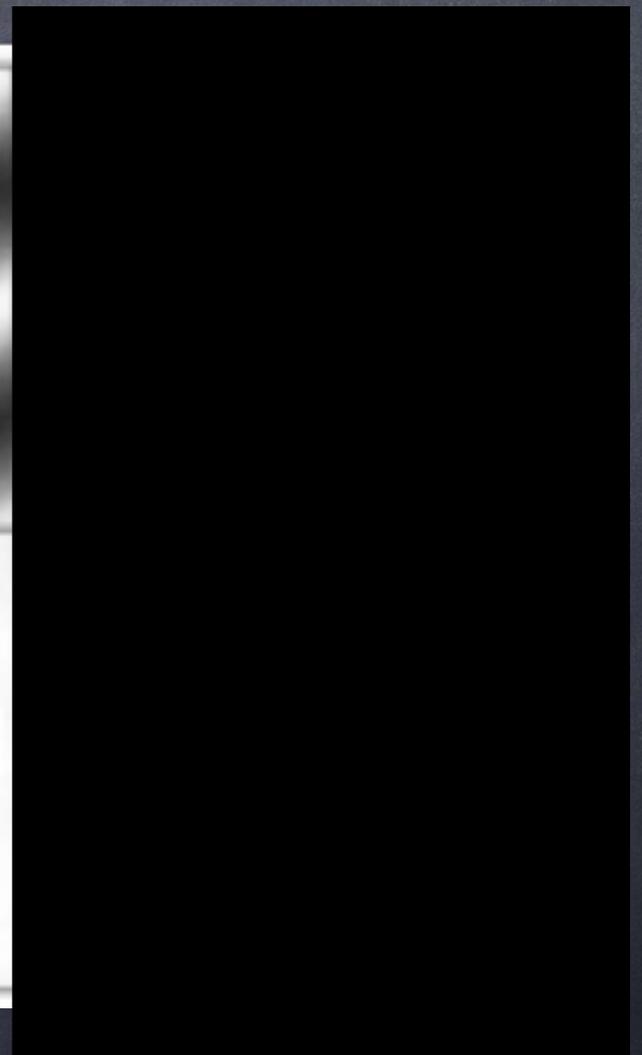
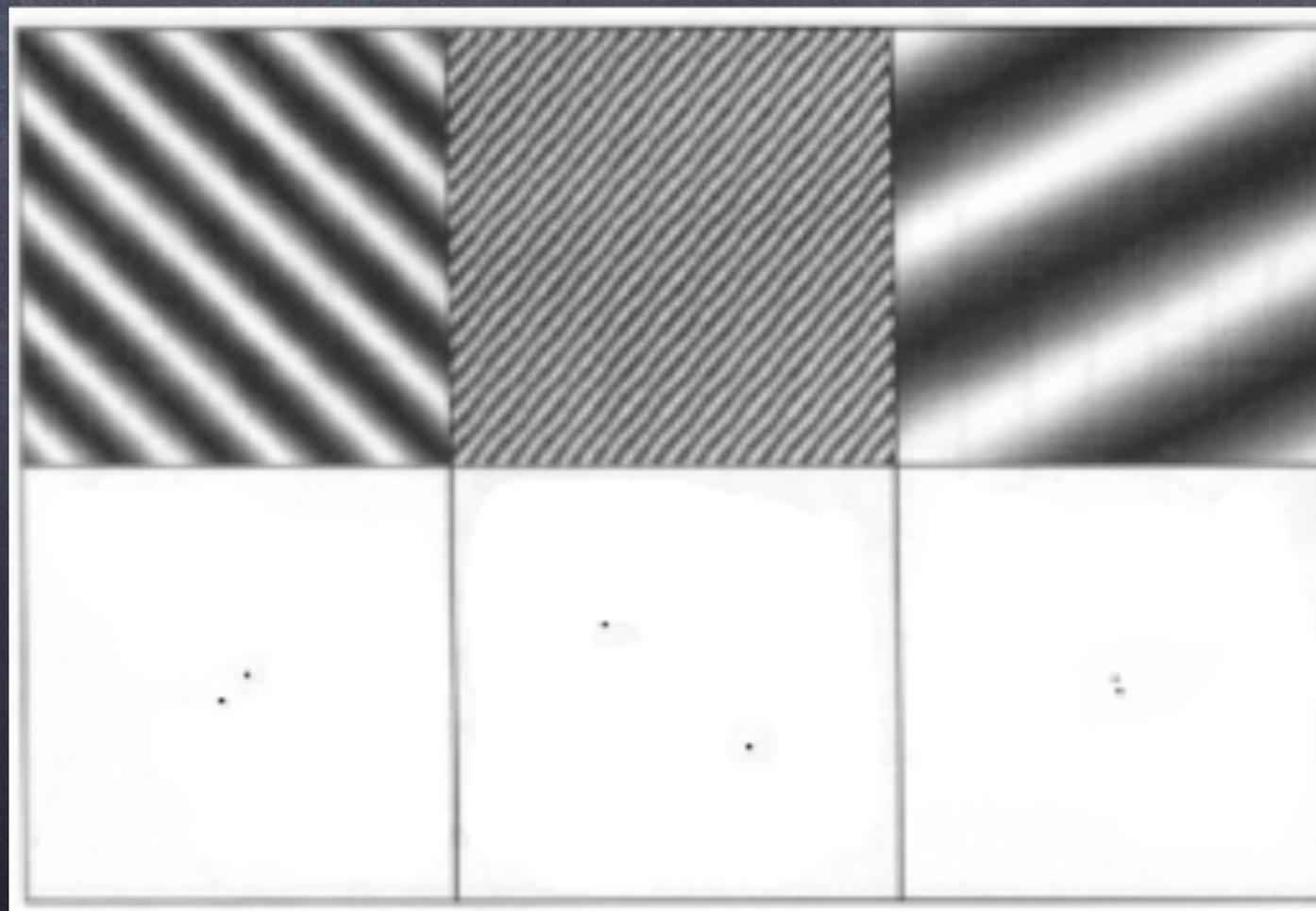


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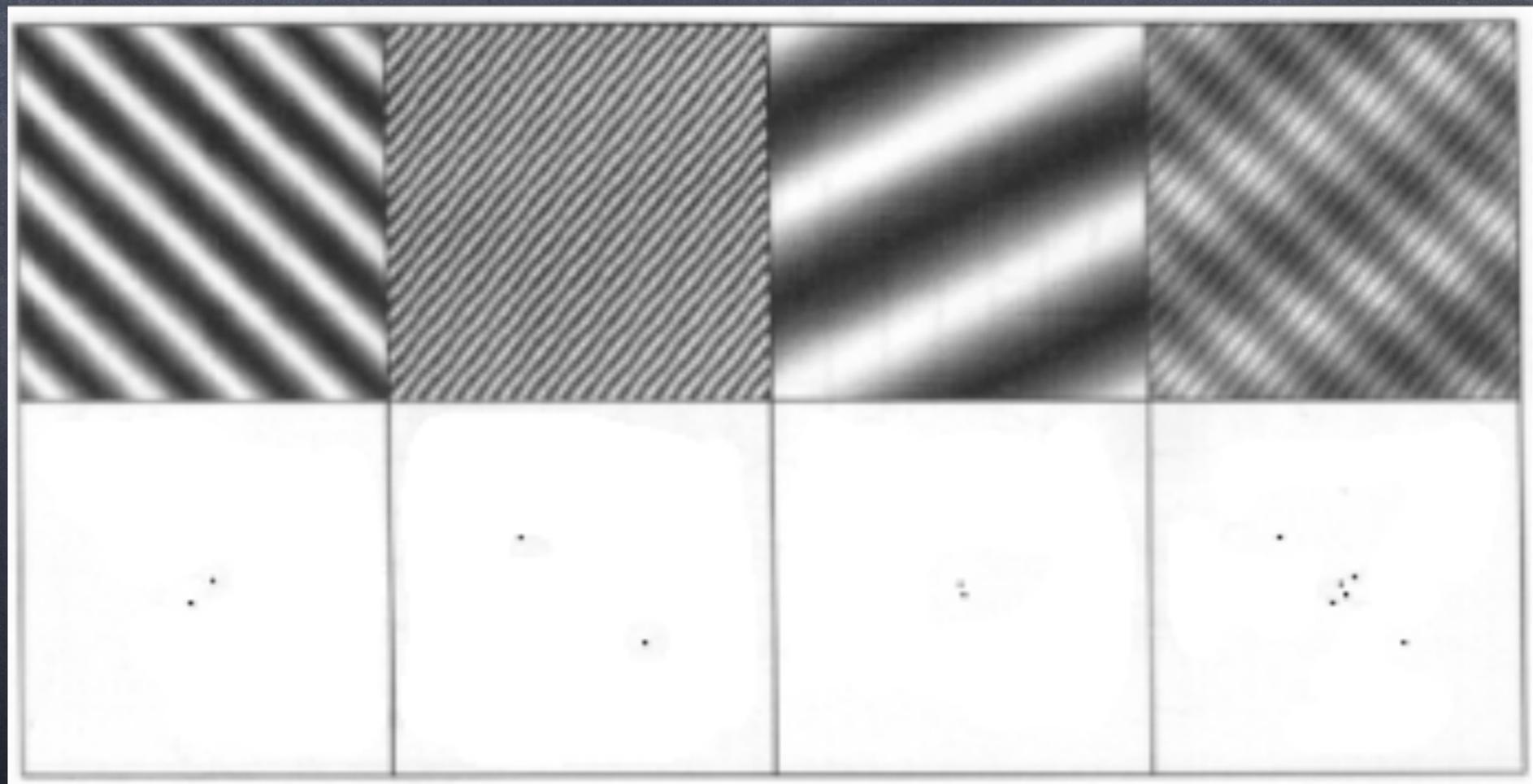
T.F.



Et l'image, elle?

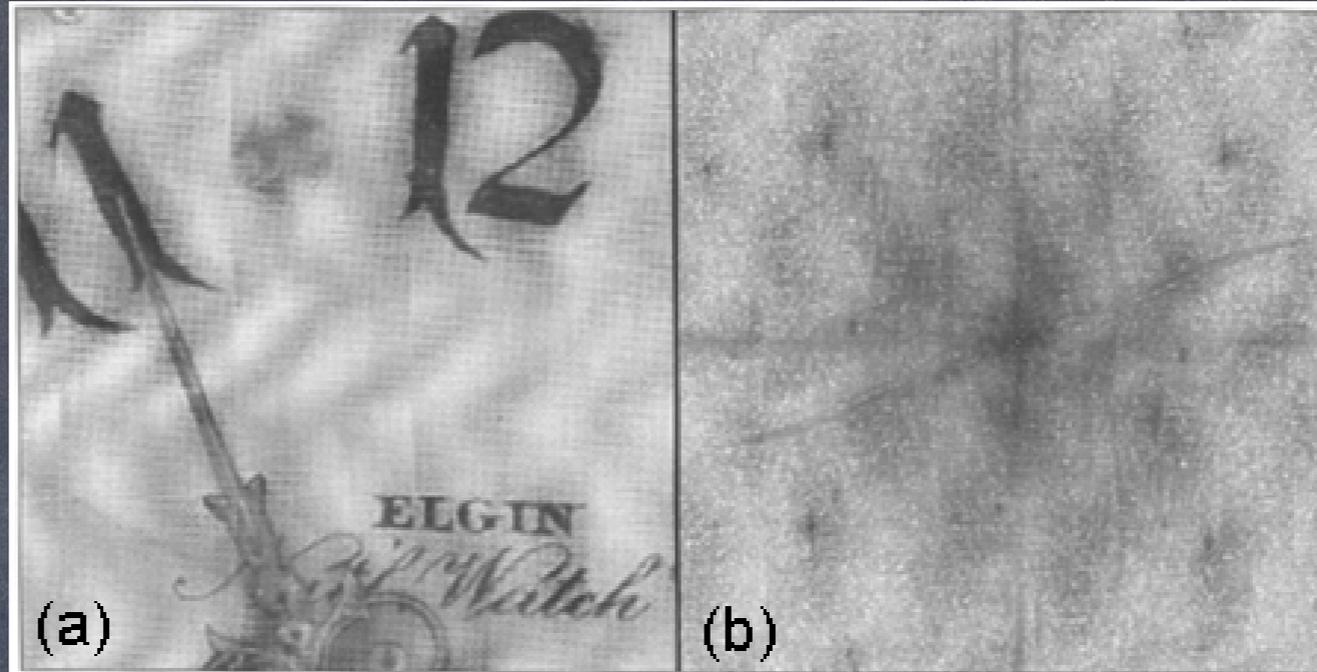
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Image



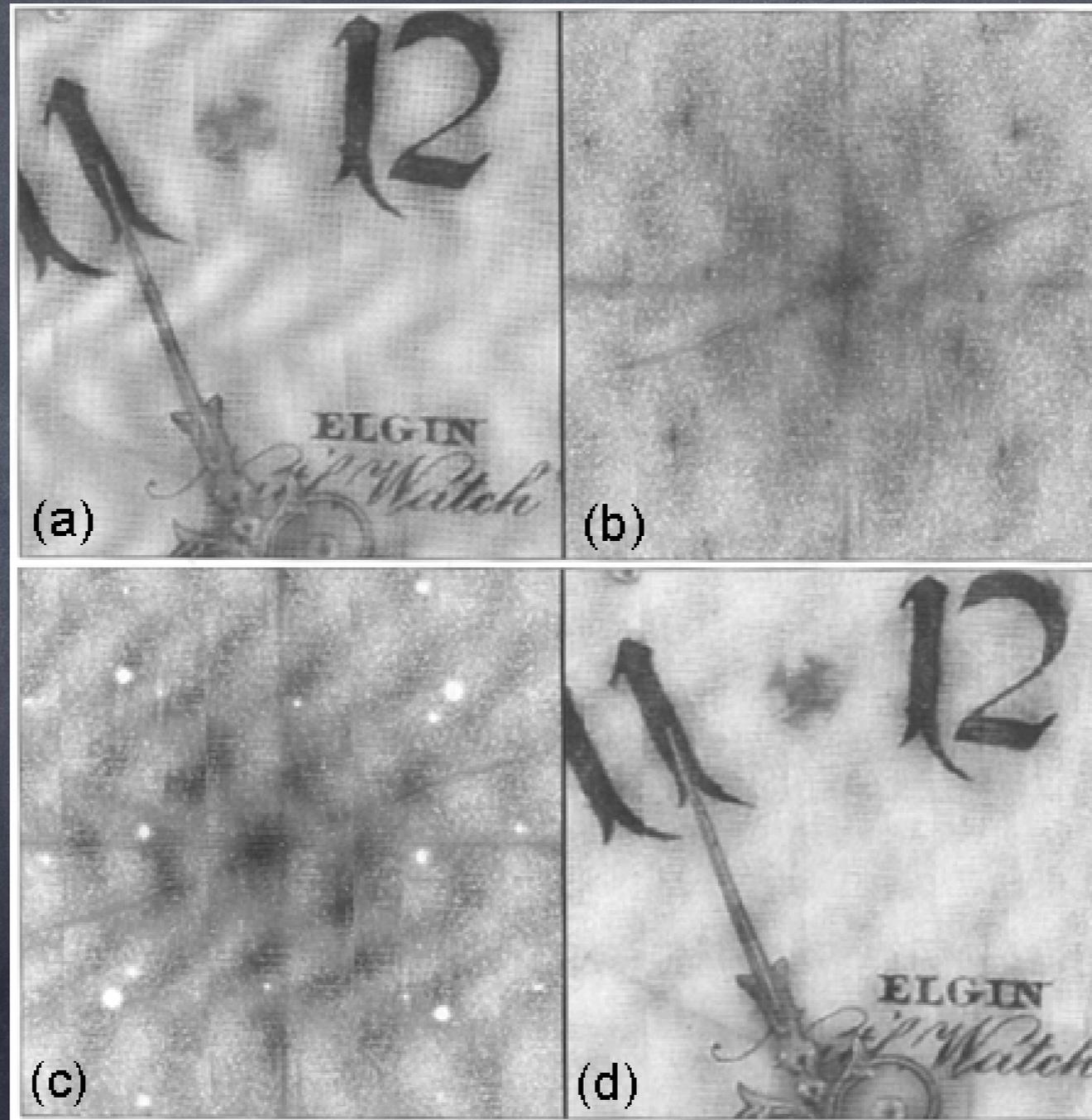
T.F.

Photocopie originale



T.F.
On voit les étoiles!

Photocopie originale

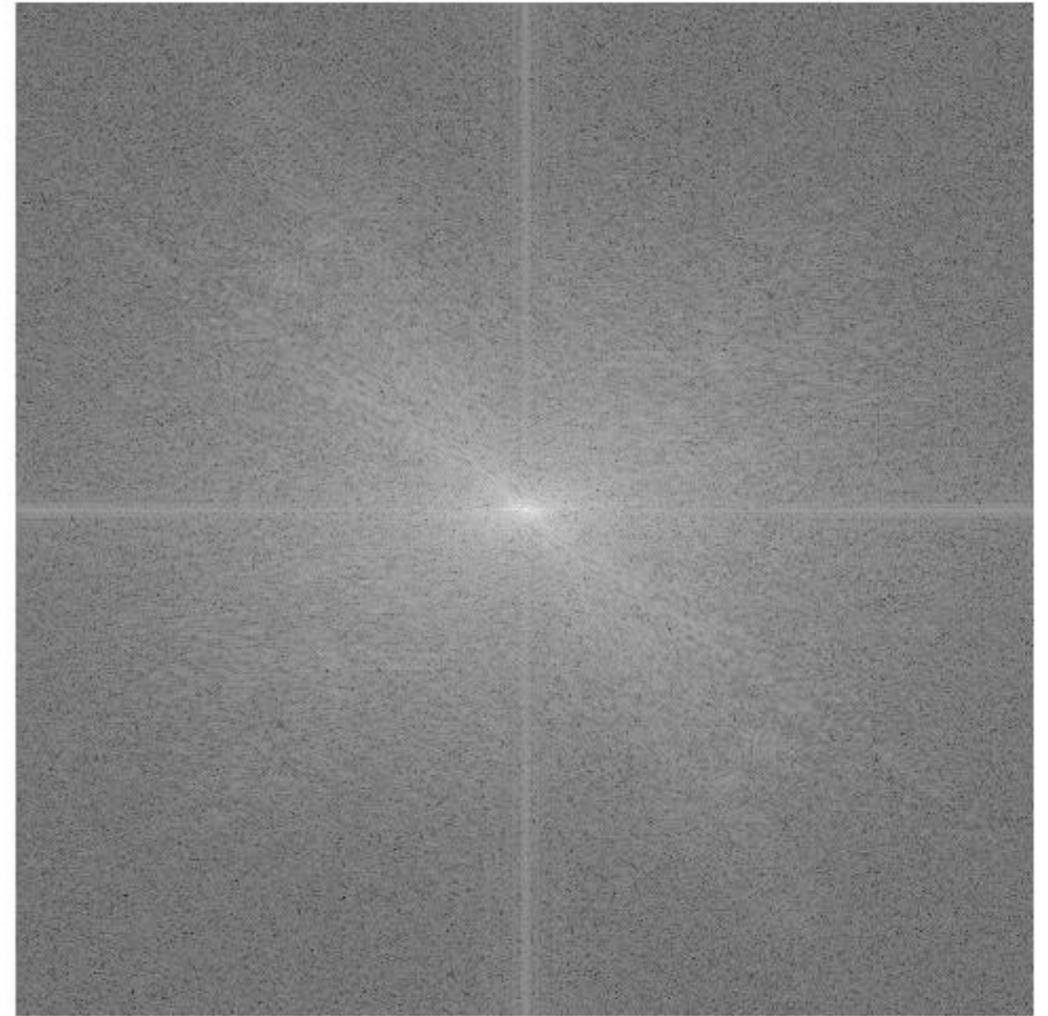


T.F.
On voit les étoiles!

F.T.
Enlevons les étoiles
(coup de gomme à effacer)

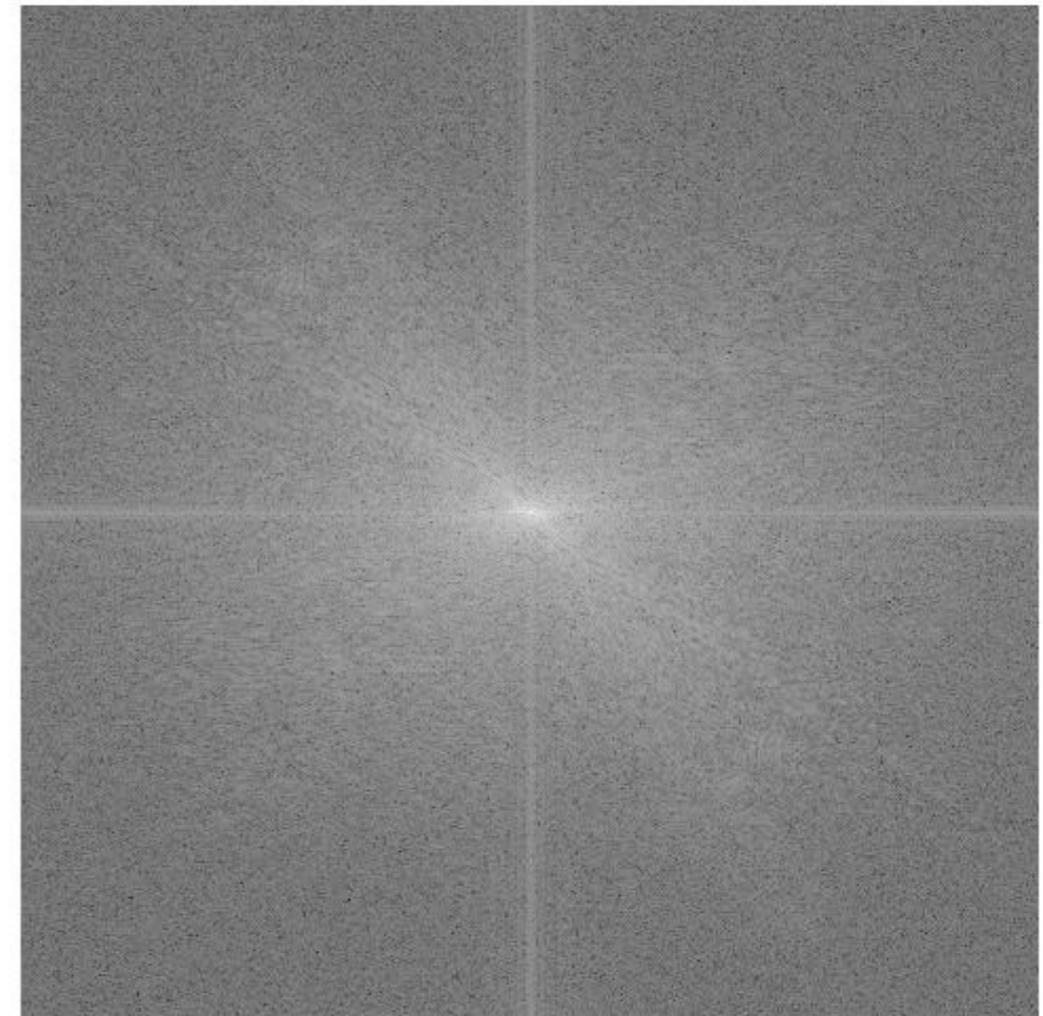
T.F.I
Wow!

TP pratique



TP pratique

La FFT2 de Lena



TF Lena (demo06)

- load
- imshow



TF Lena

- $\text{fft2}(M)$ (pleins de complexes)
- $\text{abs}(\text{fft2}(M))$ donne le module de la TF
- C'est quoi le min, max de la $\text{abs}(\text{fft2}(M))$?
 - $L = \text{abs}(\text{fft2}(M));$
 - $\text{min}()$ $\text{max}()$
 - $(0.0081, 118010)$

- imshow (on voit rien)

- On ne voit rien! (min, max) trop loins

- Le max est ou?

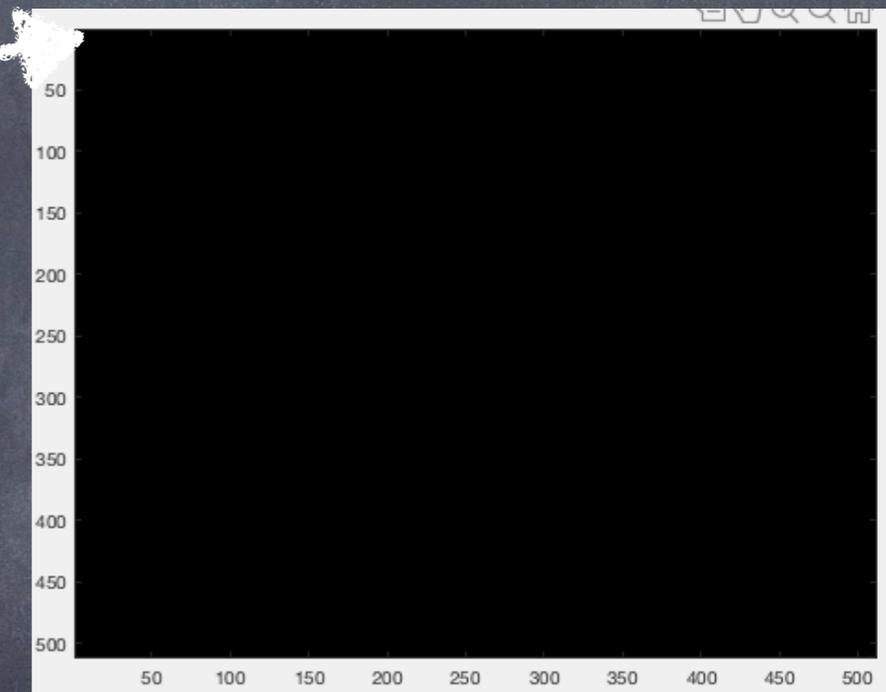
- (0,0) le coefficient_0,0

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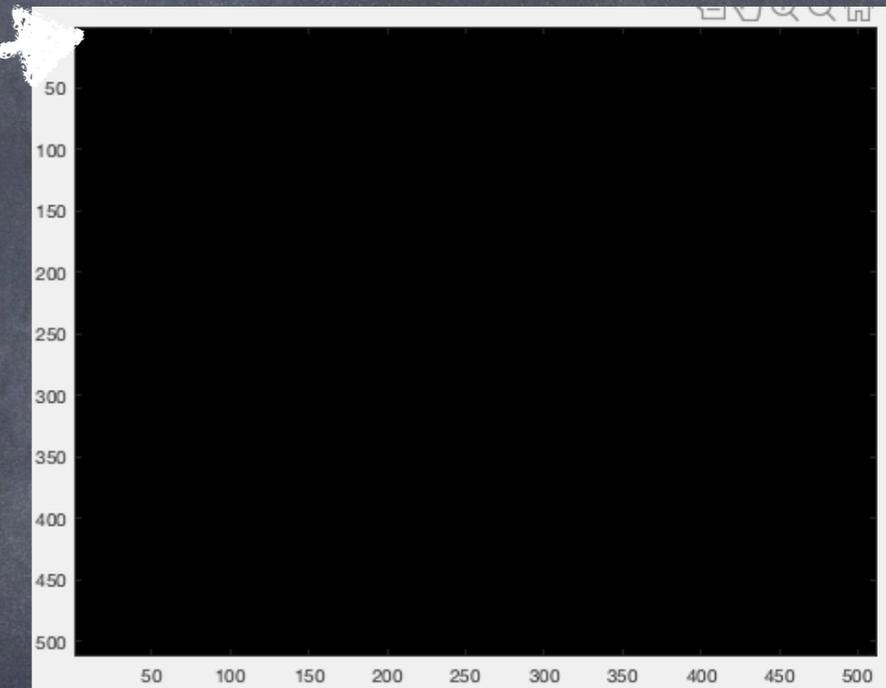
- `imshow` (on voit rien)

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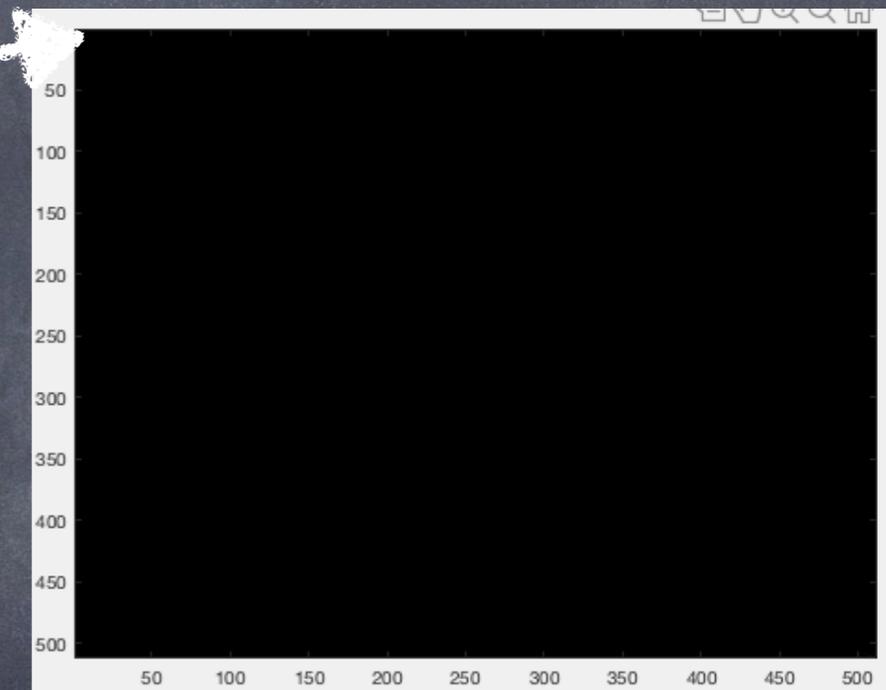
- `imagesc(fftshift(L))`



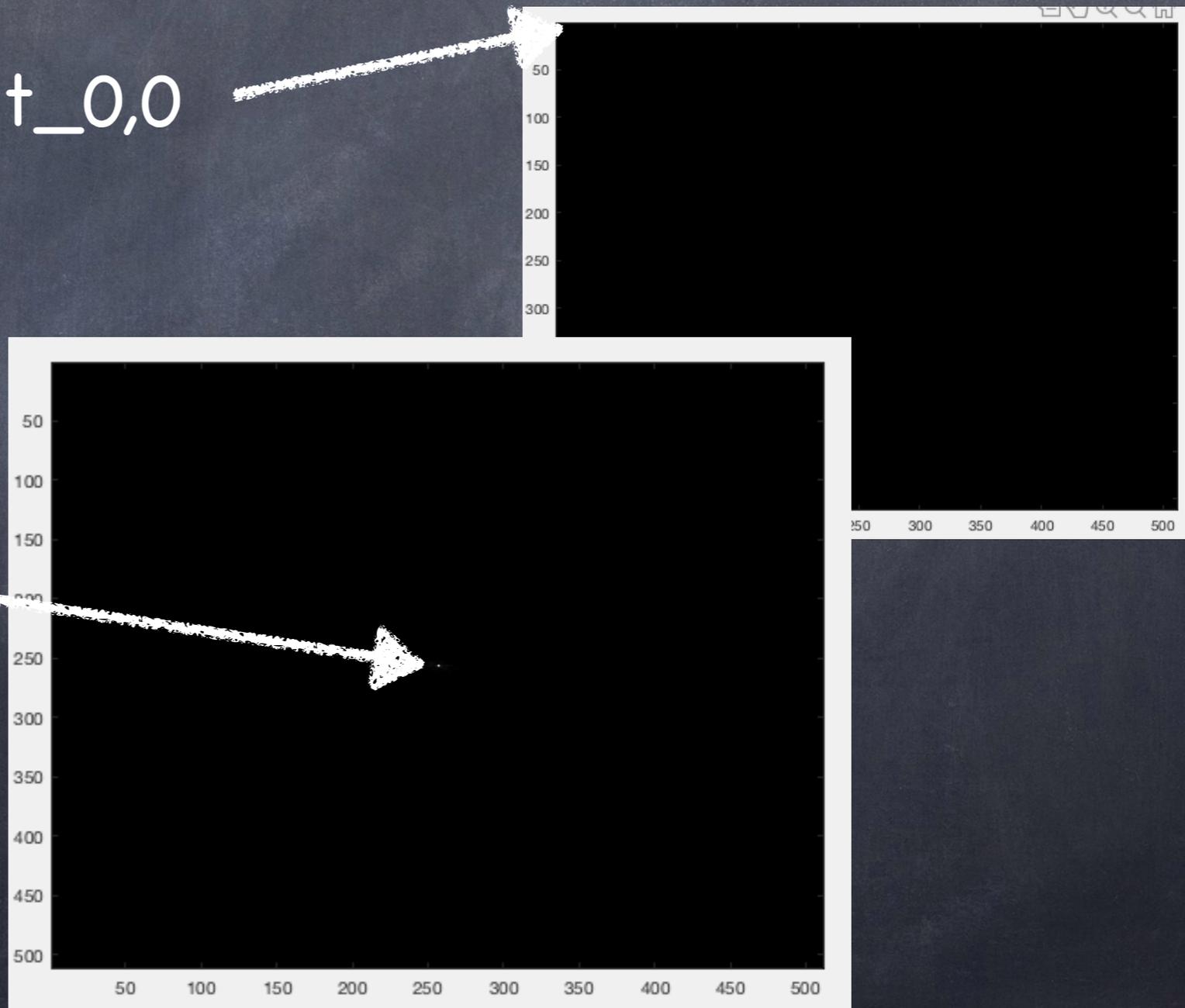
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- `L_shift = fftshift(L)`



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- `imagesc(fftshift(L))`
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- `L_shift(256,256)`



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Petit trick de visu de TF

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- On prend le log
 - $\log(118010) = 11.67$
 - $\log(0.0081) = -4.8144$

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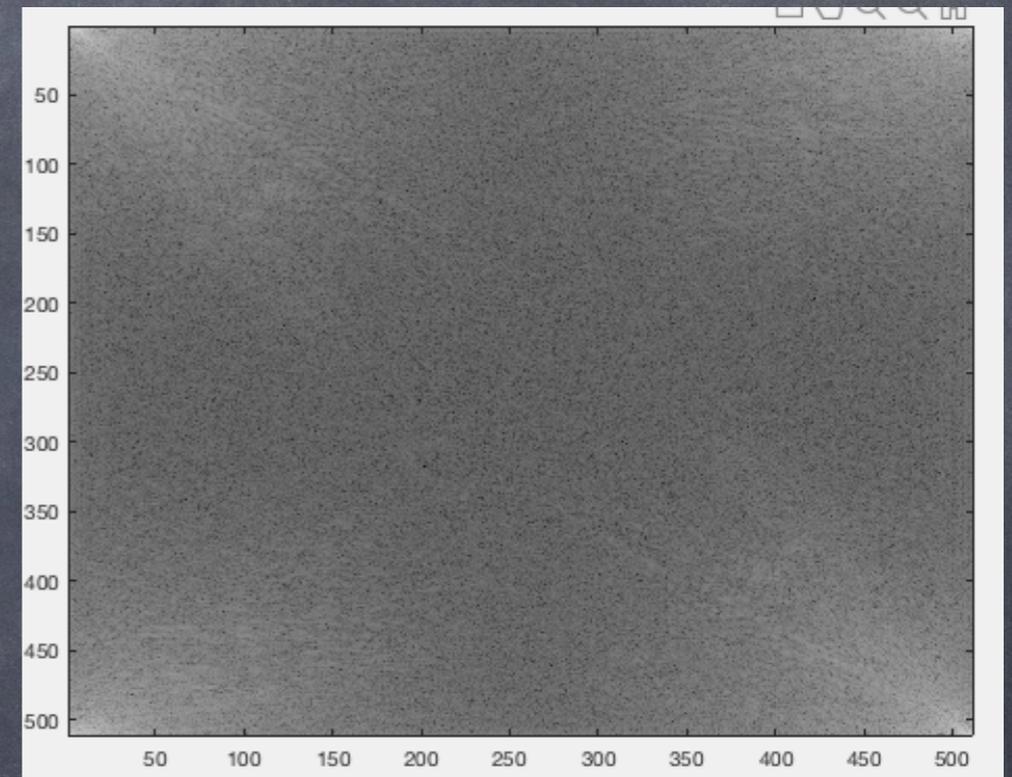
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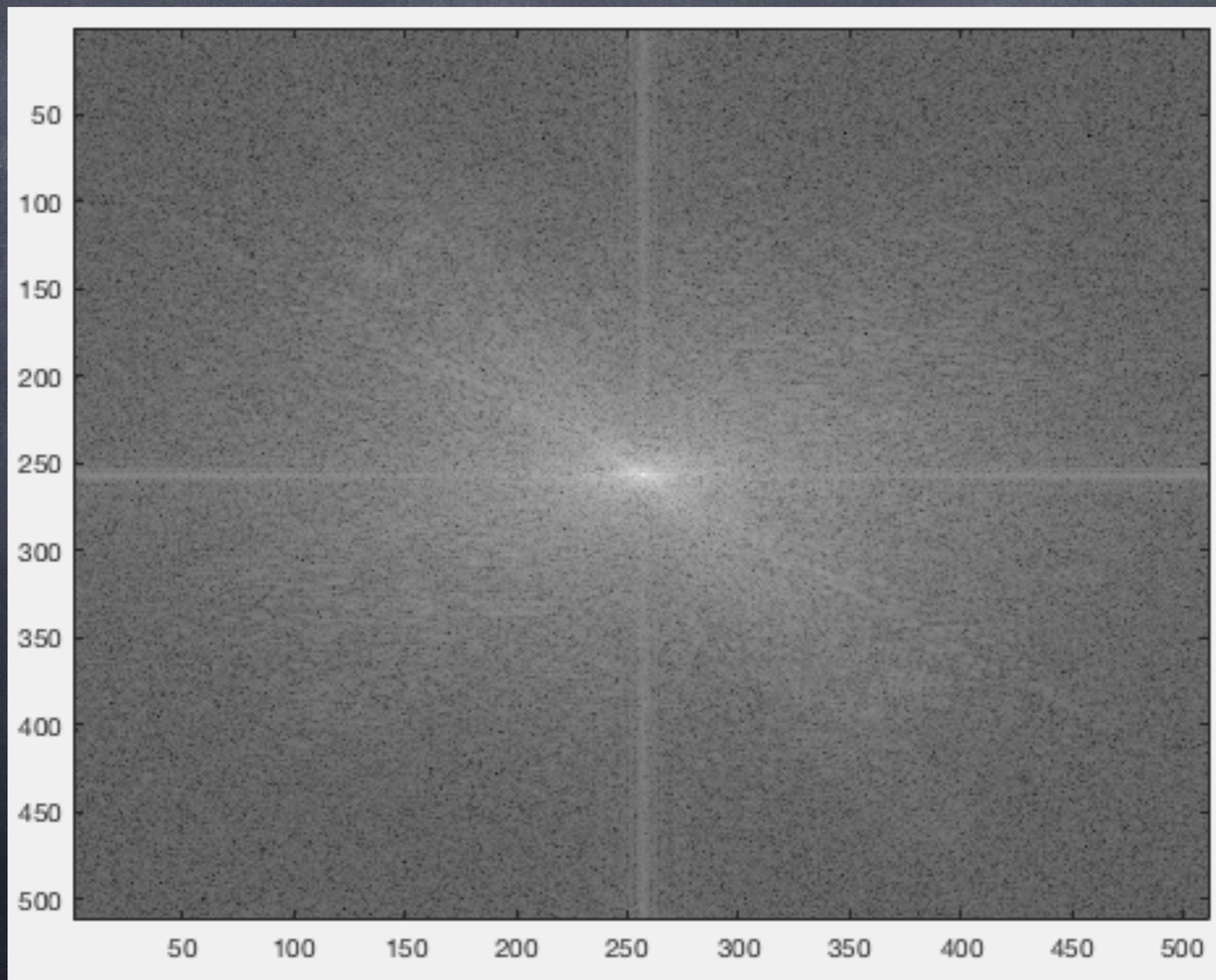
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- `imshow(log(abs(fft2(lena))))`

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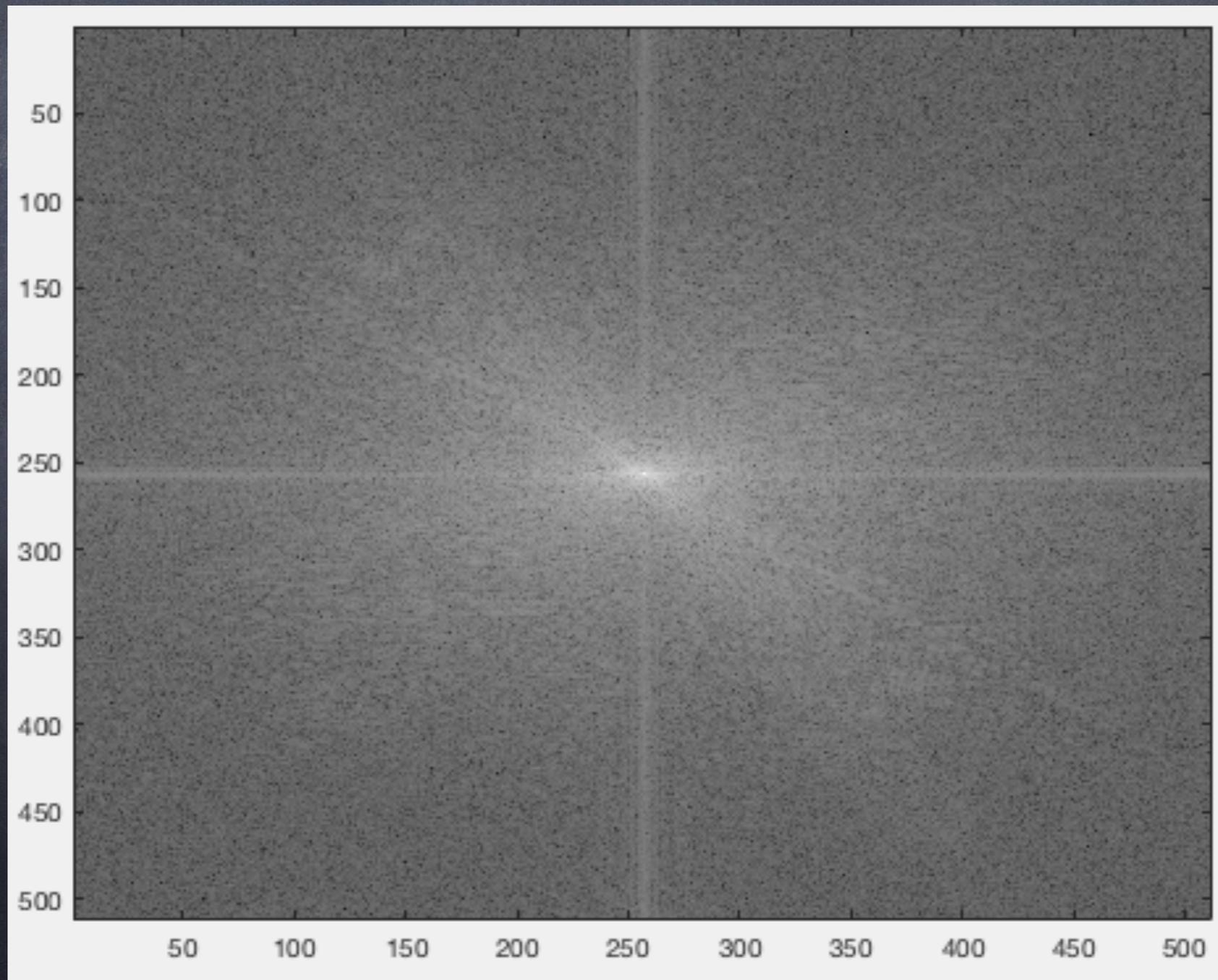


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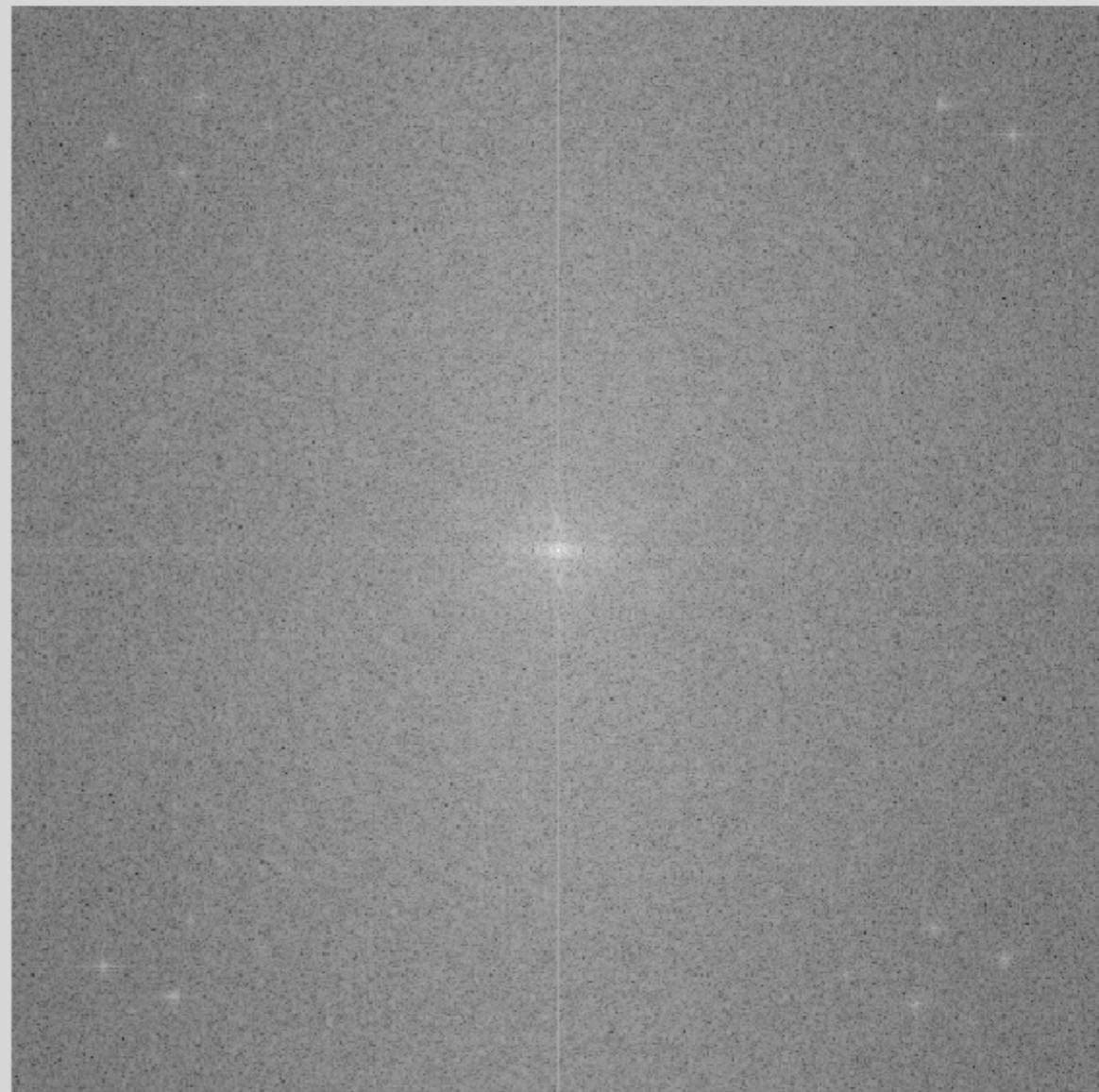
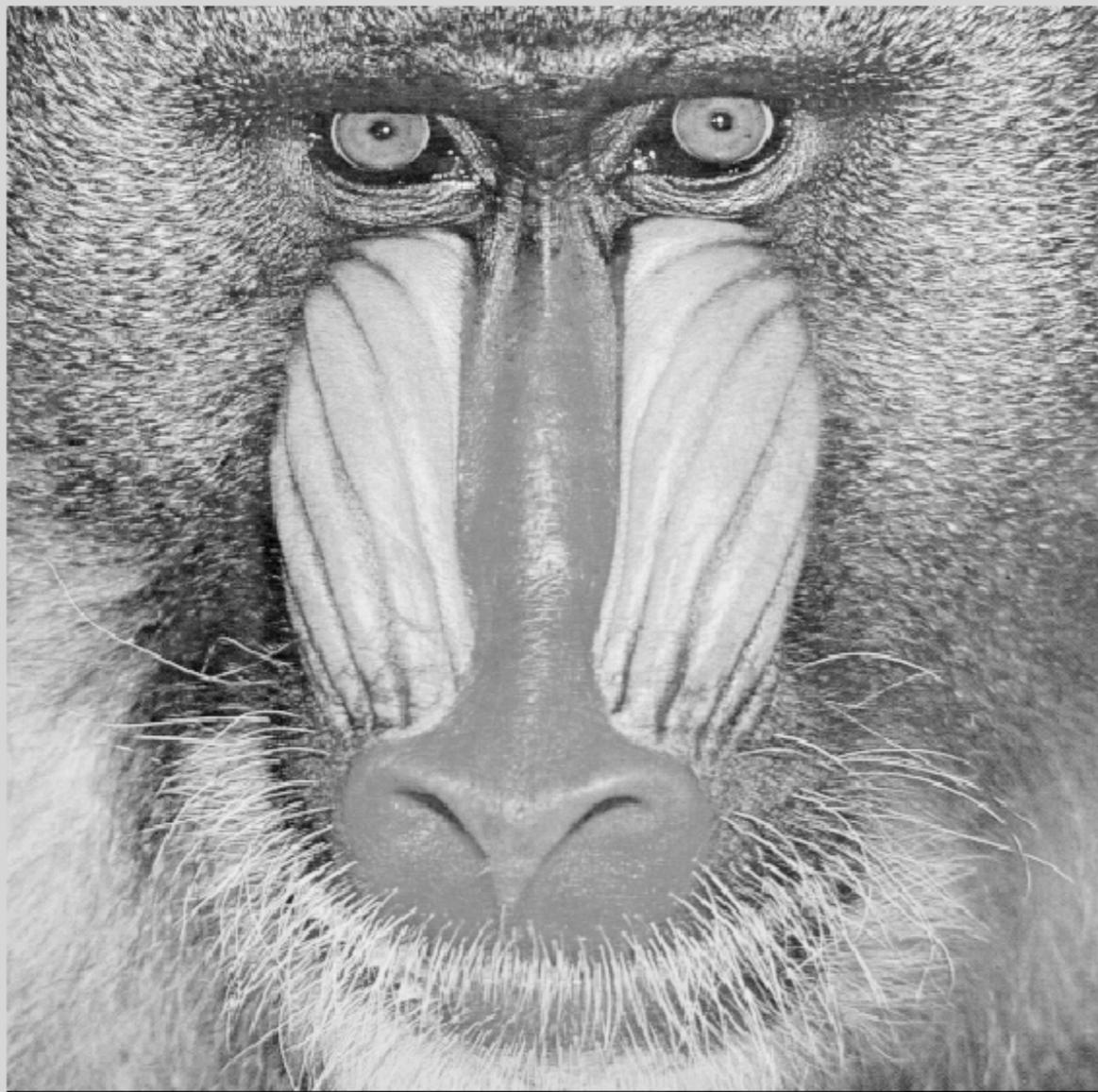


Petit trick de visu de TF

👁️ `imshow(log(abs(fftshift(fft2(lena)))))`



Mandrill



Mandrill

Cropped spectrum : 25% of coefficients

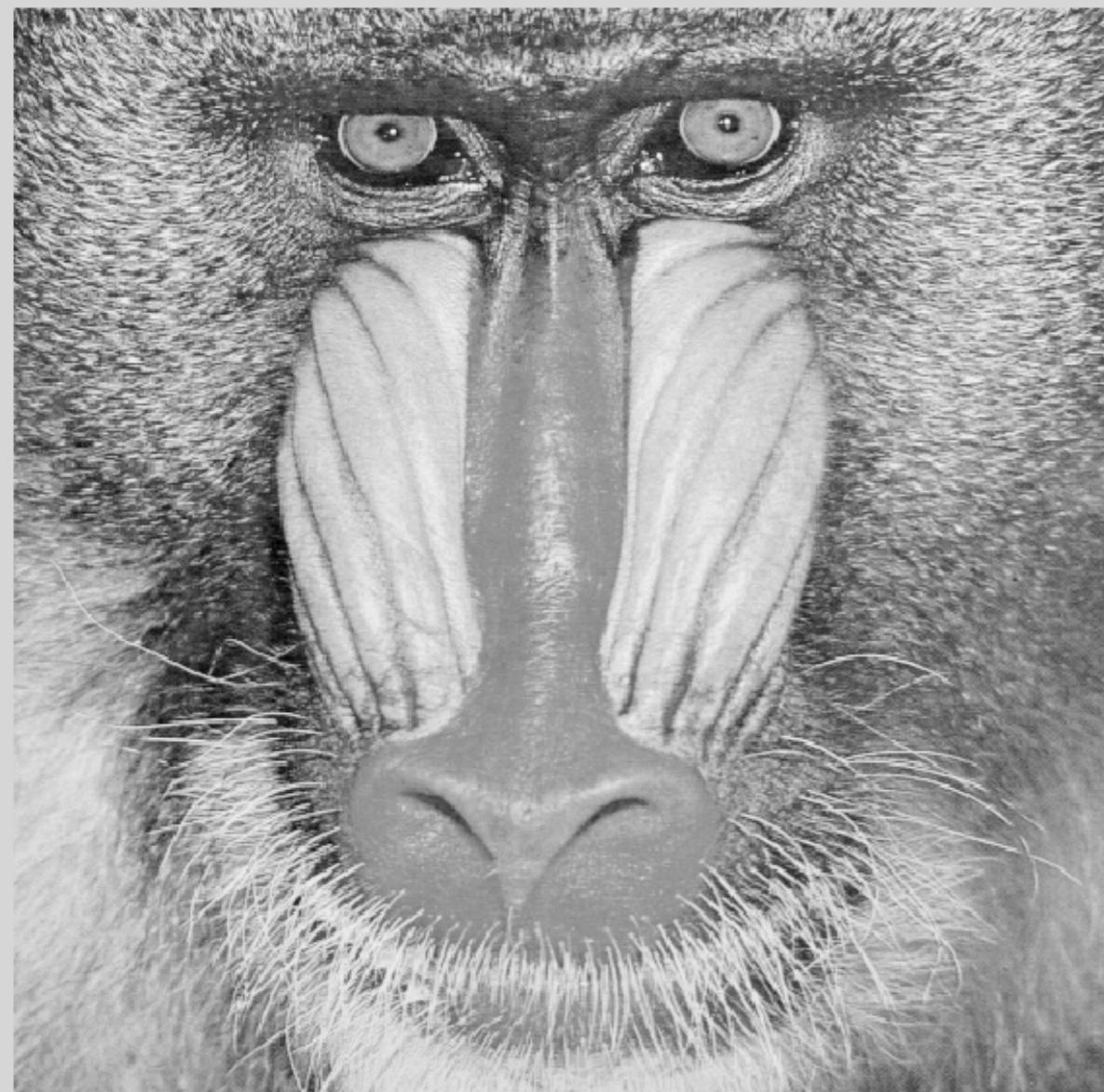


Original spectrum

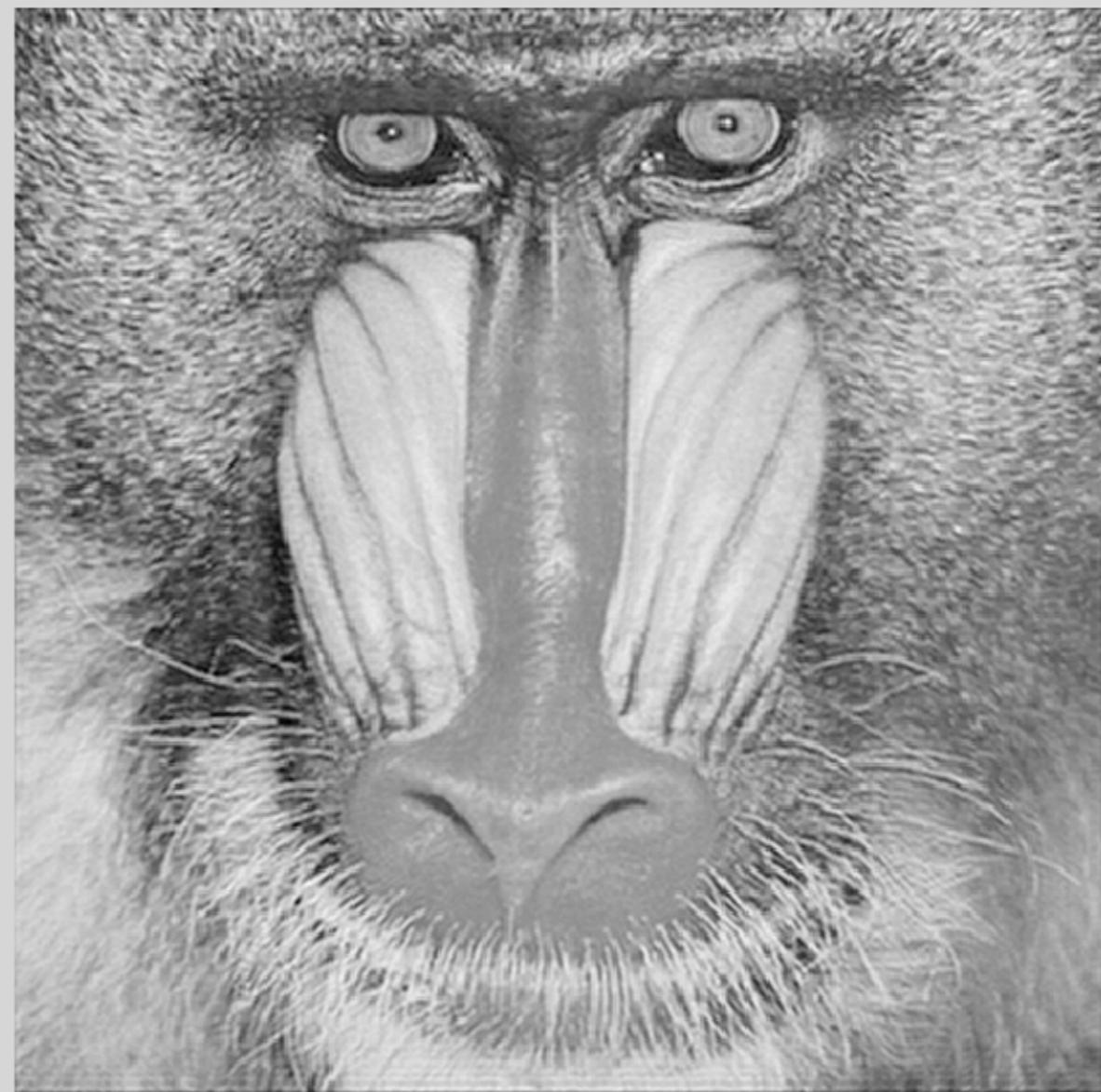


Mandrill

Original image

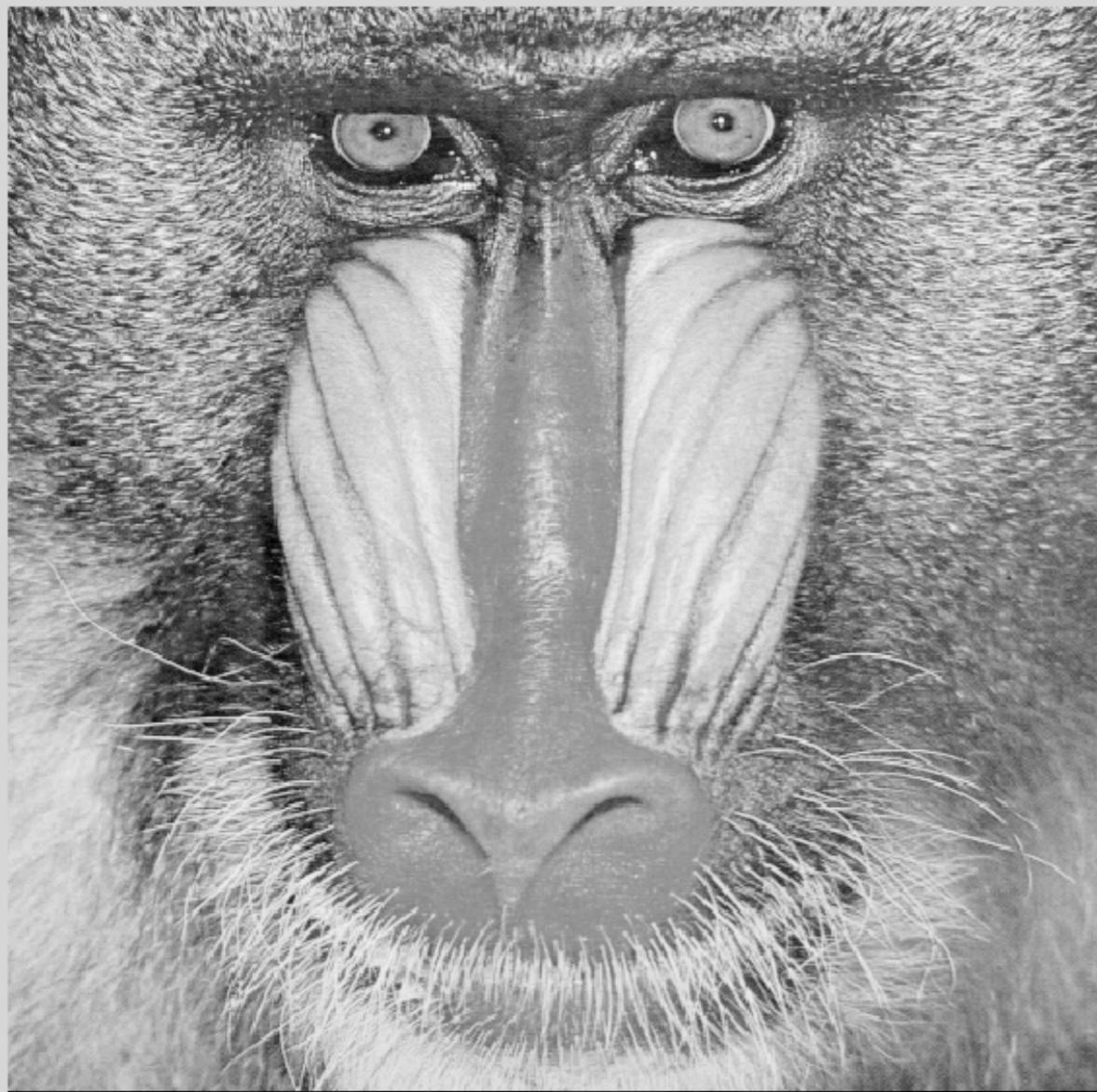


Approx, SNR=18.92dB



Mandrill

Original image

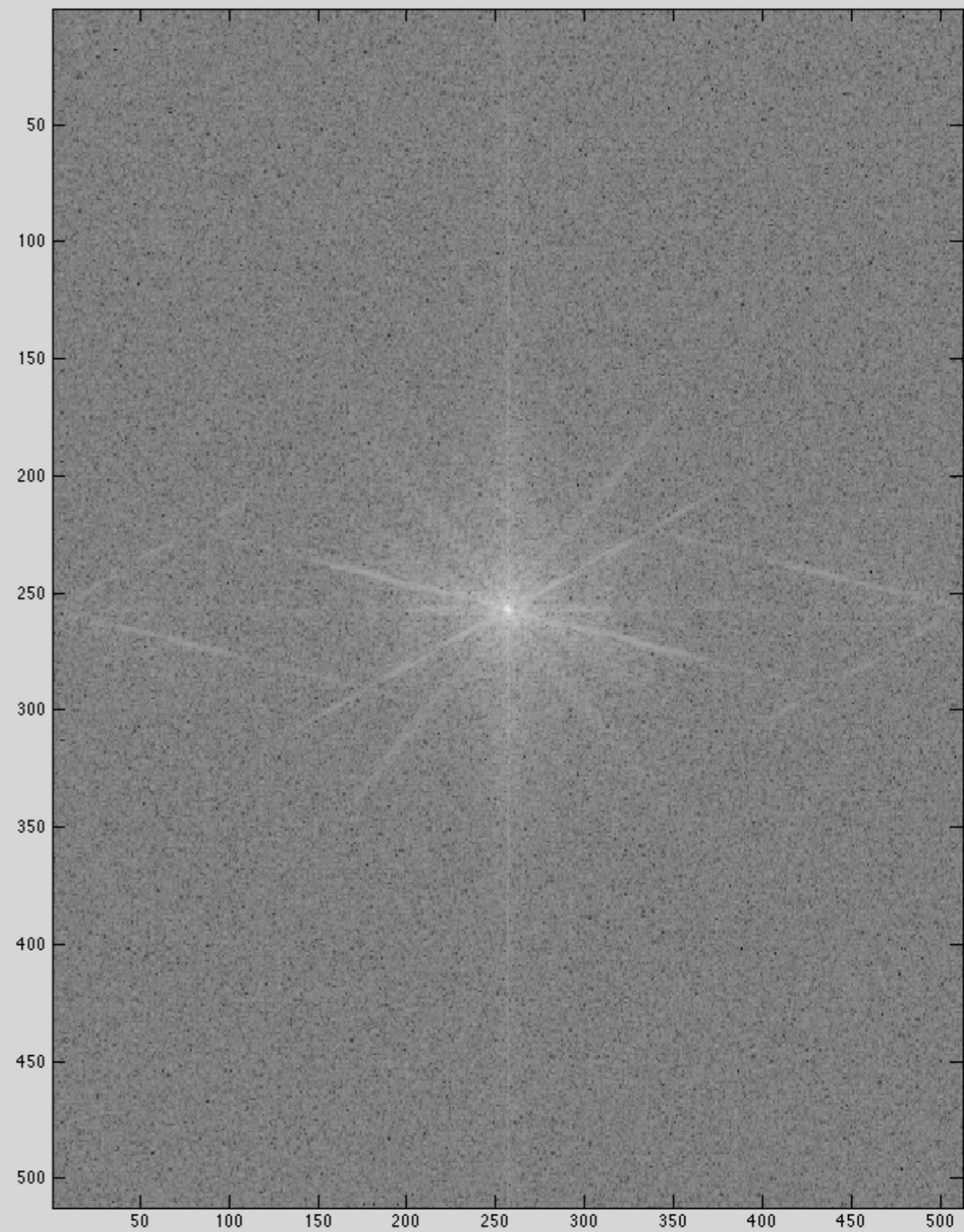


Approx, SNR=13.8dB



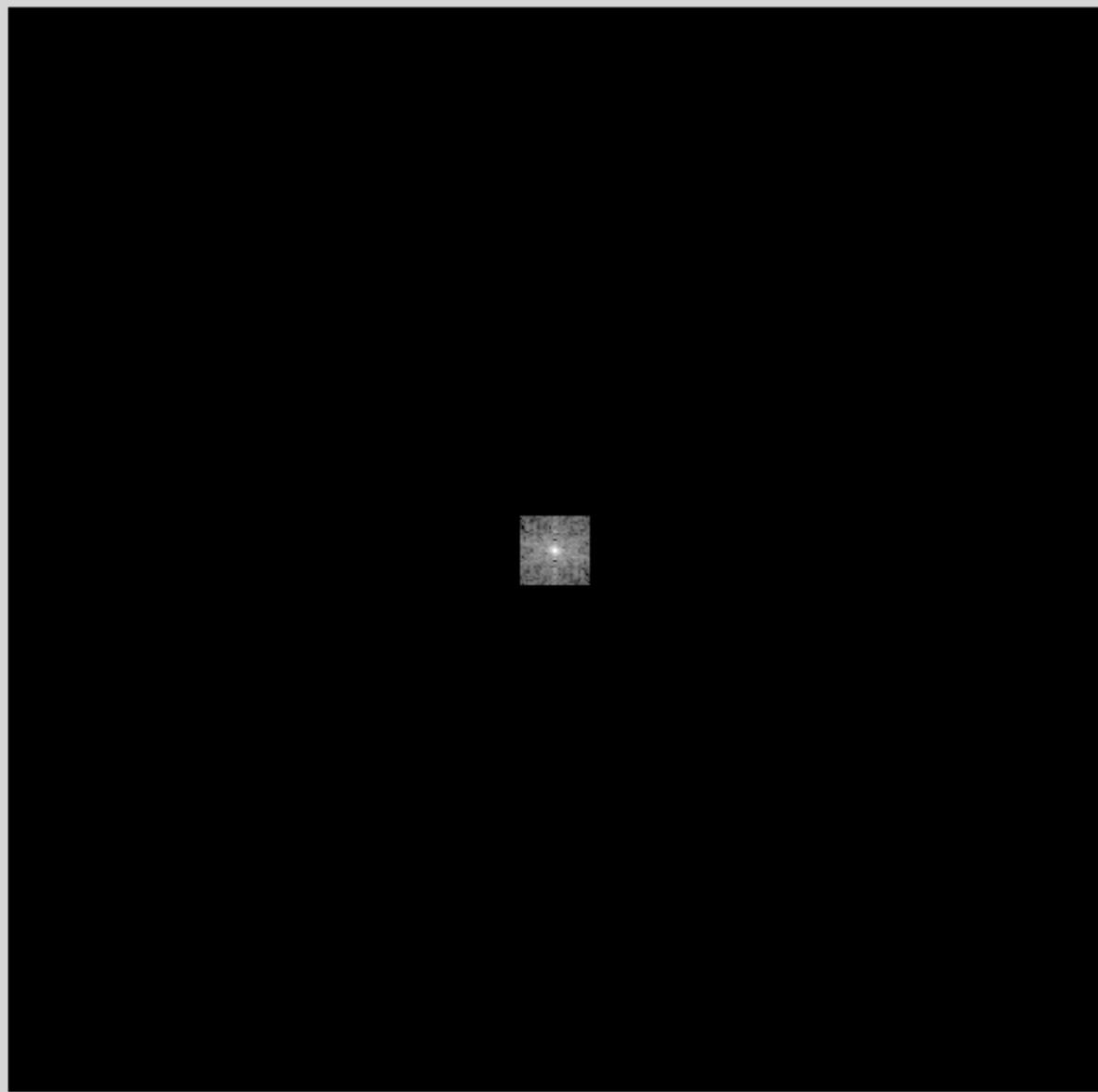
Camera Man

Original image

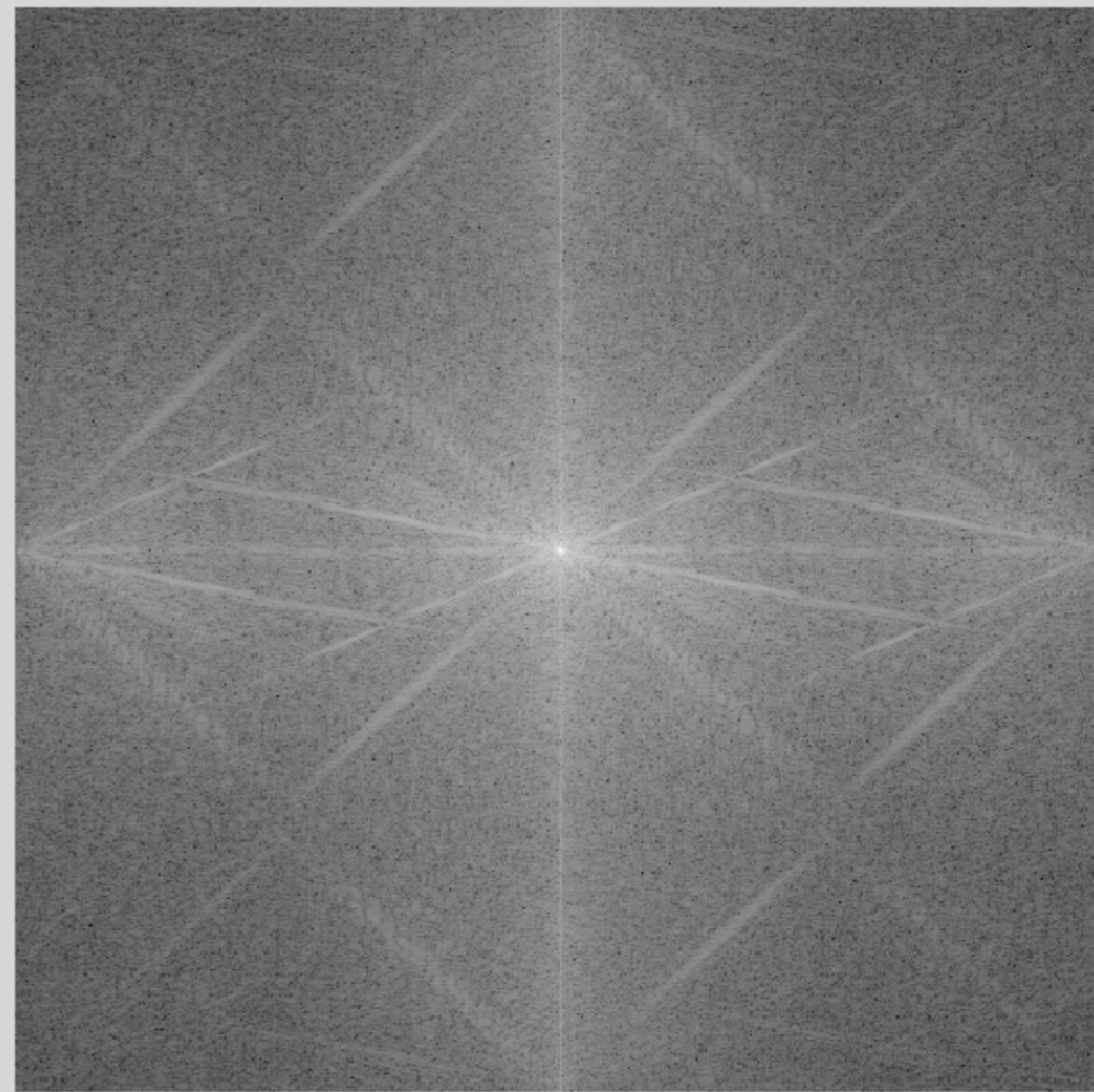


Camera Man

Cropped spectrum : 0.39062% of coefficients



Original spectrum



Cropped Fourier : 25% of coefficients



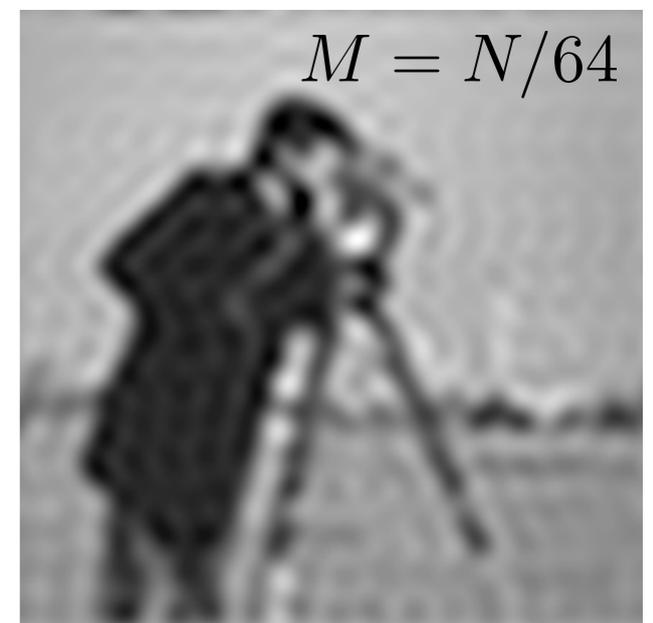
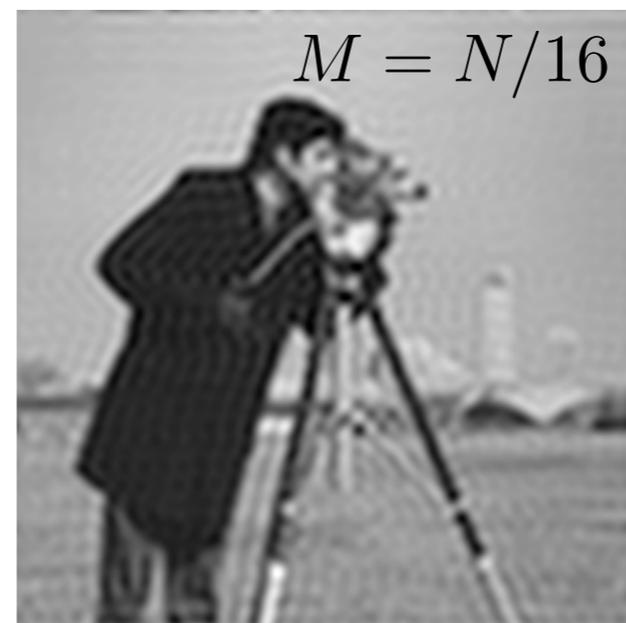
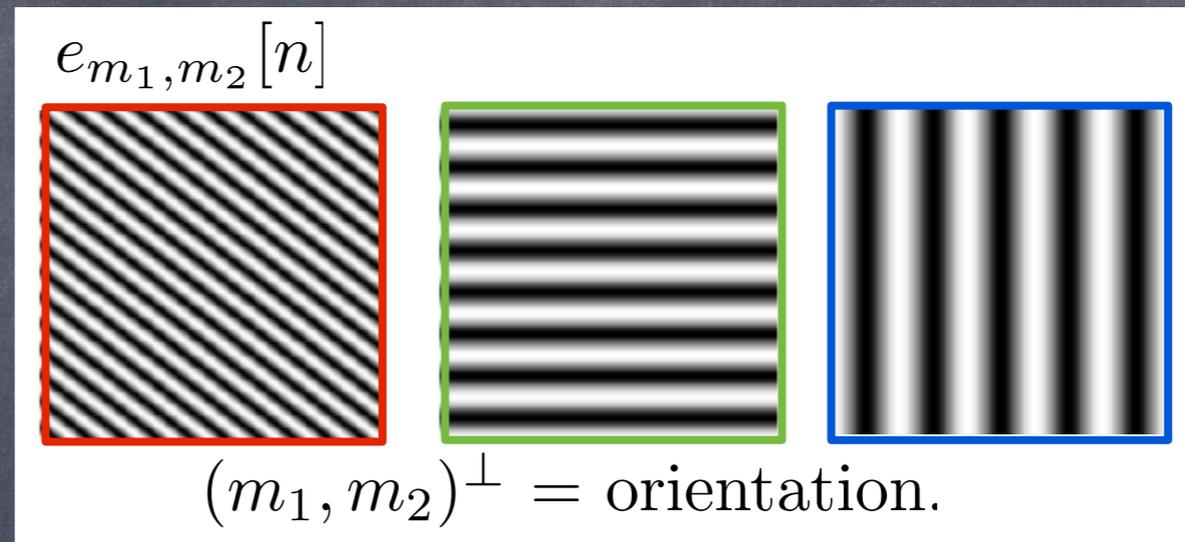
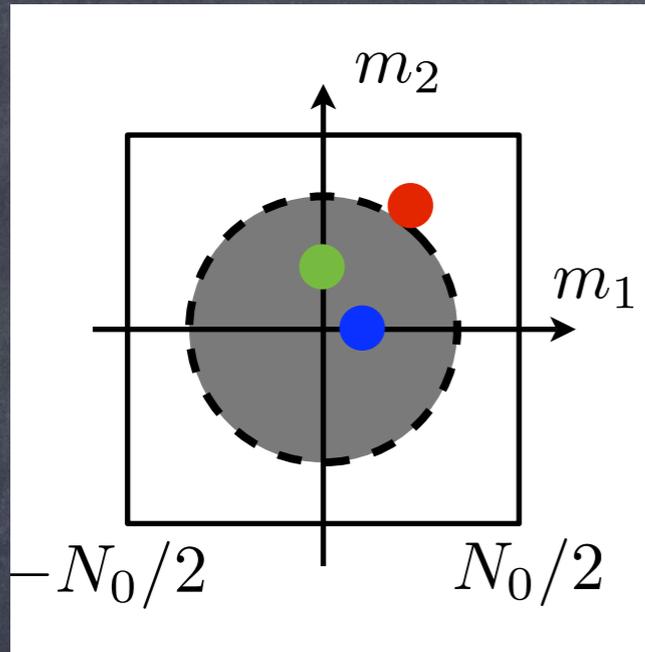
Full Fouriernoisy



Original



Fourier & discontinuities



IRM – principe

- Dispositif d'acquisition : aimant qui permet de créer des champs magnétiques

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IRM – principe

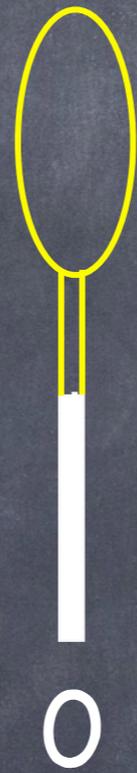
- Dispositif d'acquisition : aimant qui permet de créer des champs magnétiques
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 - Fourier inverse directement!

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- Dispositif d'acquisition : aimant qui permet de créer des champs magnétiques
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 - Fourier inverse directement!

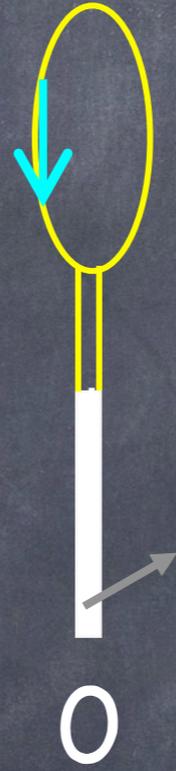
Futur: acquisition sous-échantillonnée
(compressed sensing – IMN764)

Courant



Temps

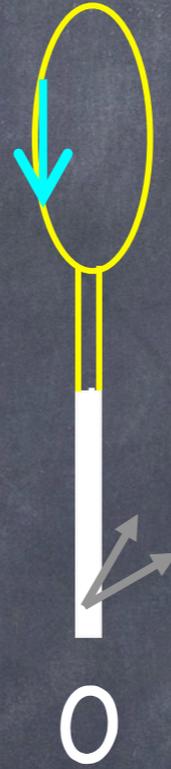
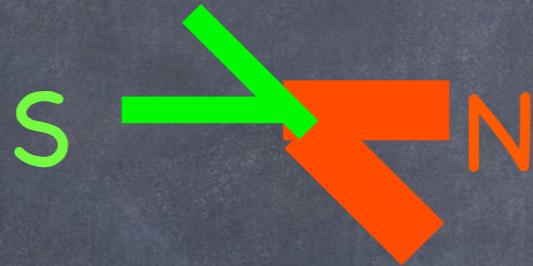




Courant



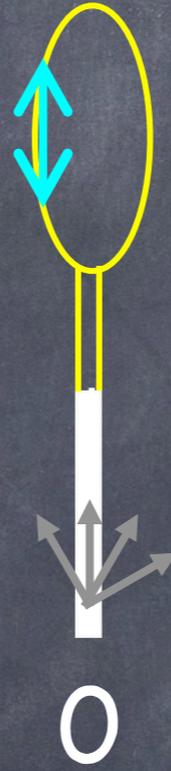
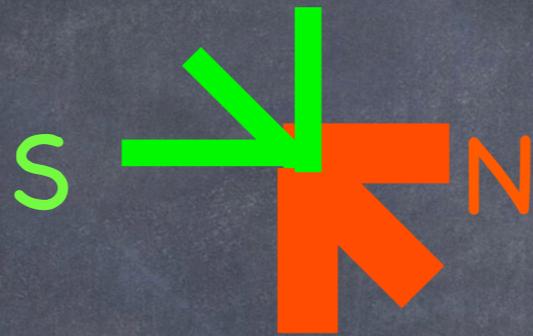
Temps



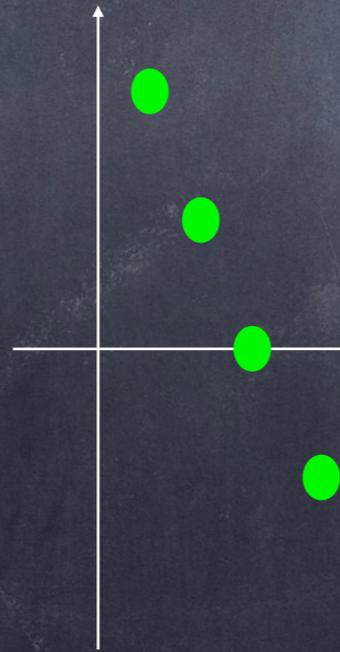
Courant



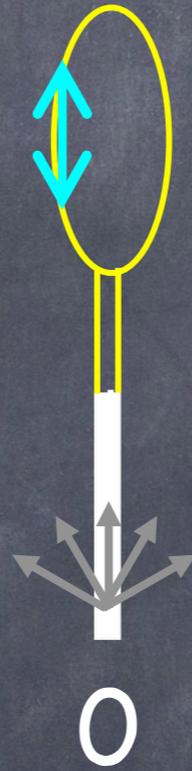
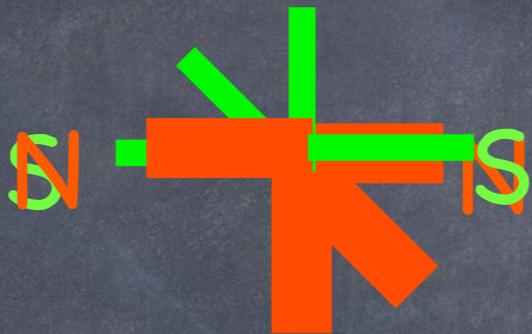
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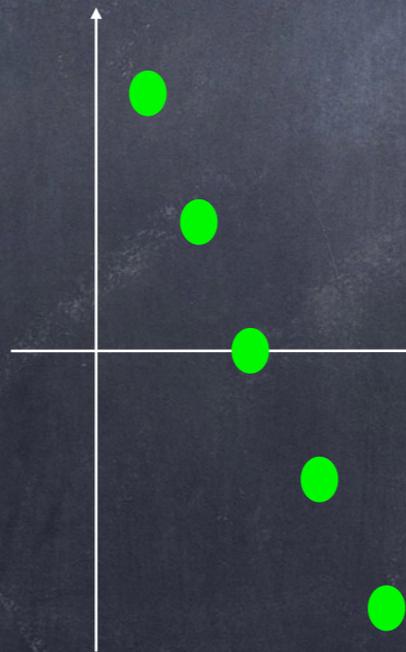
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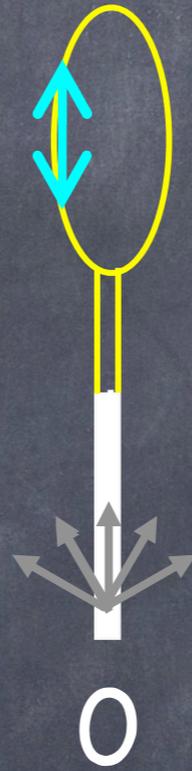
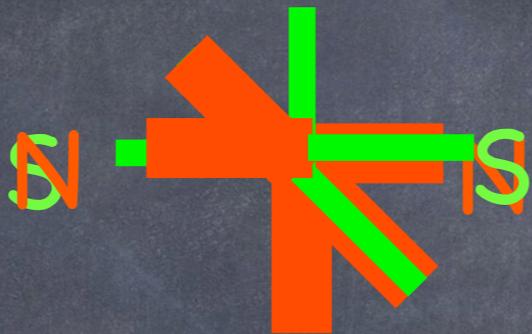
Temps



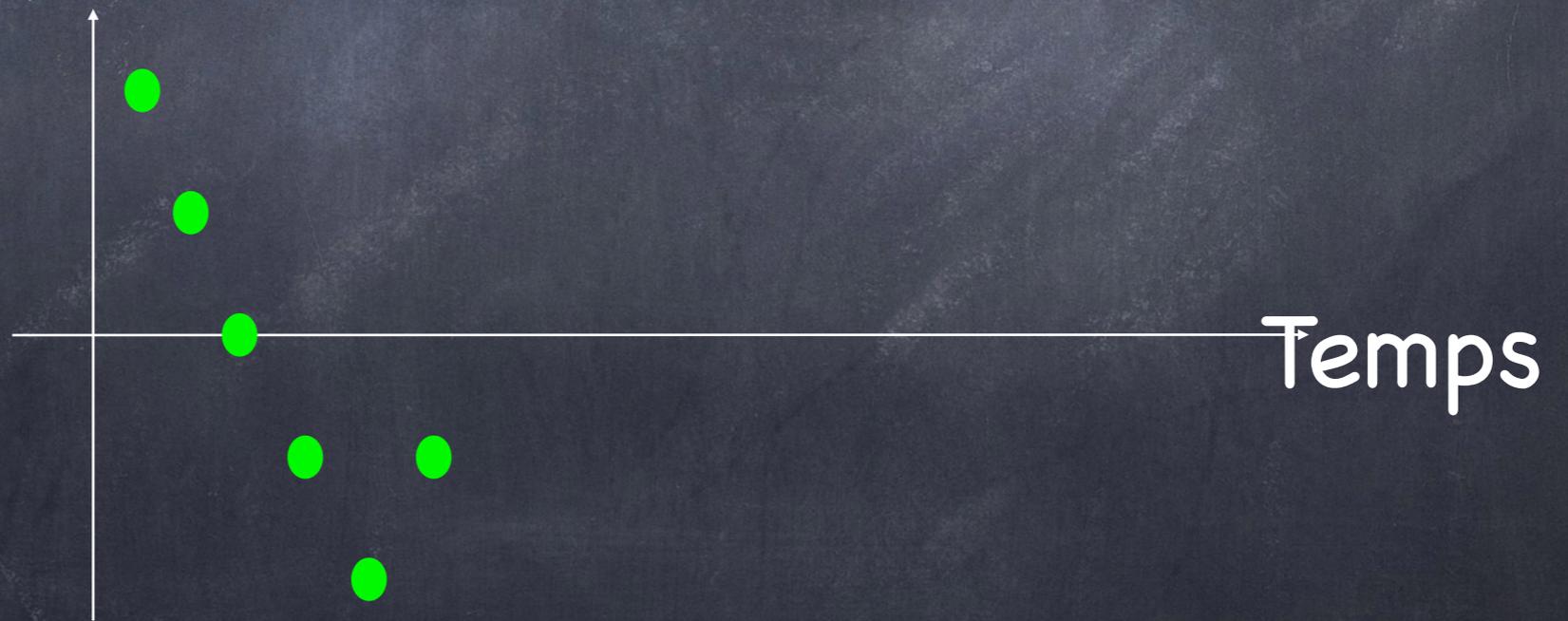
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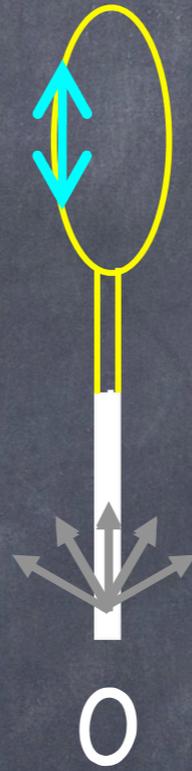
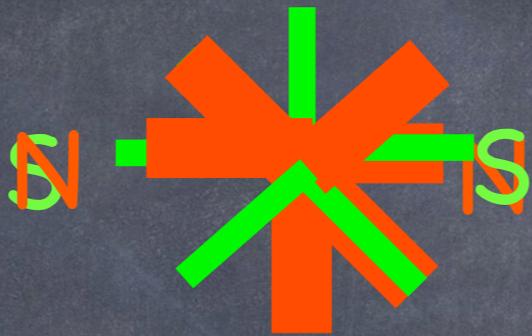


Temps

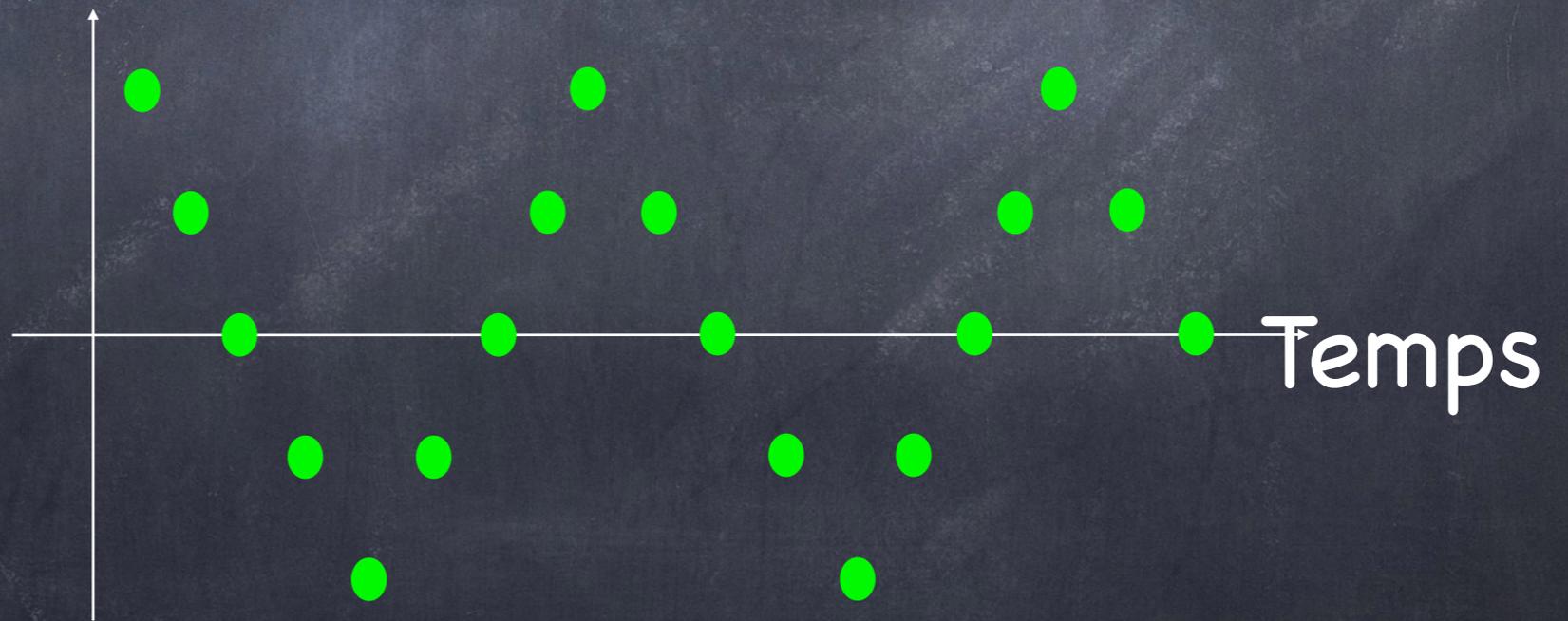


Courant





Courant



Collecte de l'information fréquentielle

| | | | | | | | | | | | | | | | | | | | | | | | | | | |
|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|
| 175 | 175 | 176 | 175 | 175 | 171 | 166 | 164 | 163 | 161 | 158 | 157 | 155 | 154 | 150 | 147 | 143 | 141 | 139 | 136 | 134 | 131 | 128 | 125 | 122 | 120 | 118 |
| 178 | 178 | 178 | 178 | 177 | 174 | 173 | 168 | 167 | 164 | 162 | 160 | 157 | 156 | 153 | 151 | 147 | 145 | 143 | 140 | 138 | 135 | 131 | 127 | 125 | 125 | 122 |
| 180 | 180 | 180 | 180 | 180 | 178 | 176 | 173 | 171 | 167 | 167 | 164 | 162 | 160 | 156 | 155 | 151 | 149 | 146 | 143 | 140 | 139 | 136 | 132 | 130 | 128 | 127 |
| 185 | 184 | 184 | 184 | 182 | 180 | 179 | 177 | 175 | 171 | 170 | 167 | 165 | 163 | 160 | 160 | 155 | 153 | 151 | 148 | 145 | 142 | 139 | 137 | 136 | 133 | 129 |
| 194 | 194 | 191 | 187 | 185 | 185 | 183 | 182 | 180 | 176 | 175 | 174 | 170 | 170 | 153 | 163 | 161 | 158 | 155 | 152 | 150 | 147 | 144 | 141 | 139 | 136 | 136 |
| 199 | 199 | 197 | 194 | 192 | 188 | 185 | 184 | 183 | 180 | 178 | 176 | 173 | 171 | 145 | 165 | 163 | 160 | 157 | 155 | 152 | 149 | 147 | 144 | 141 | 139 | 137 |
| 202 | 201 | 203 | 199 | 199 | 195 | 192 | 188 | 187 | 184 | 183 | 180 | 177 | 176 | 116 | 171 | 167 | 166 | 161 | 159 | 158 | 155 | 153 | 149 | 146 | 143 | 141 |
| 206 | 205 | 205 | 204 | 201 | 200 | 198 | 197 | 183 | 144 | 169 | 184 | 177 | 178 | 81 | 174 | 172 | 168 | 160 | 164 | 162 | 159 | 156 | 153 | 151 | 149 | 145 |
| 209 | 210 | 211 | 209 | 207 | 205 | 204 | 205 | 151 | 72 | 97 | 123 | 89 | 156 | 72 | 173 | 169 | 157 | 140 | 172 | 169 | 167 | 163 | 160 | 157 | 154 | 151 |
| 215 | 215 | 213 | 213 | 213 | 209 | 209 | 208 | 147 | 65 | 86 | 98 | 66 | 76 | 79 | 169 | 165 | 122 | 85 | 154 | 174 | 172 | 169 | 166 | 163 | 160 | 158 |
| 225 | 225 | 223 | 220 | 219 | 217 | 215 | 214 | 127 | 50 | 60 | 88 | 50 | 27 | 27 | 85 | 108 | 83 | 57 | 127 | 178 | 178 | 175 | 171 | 168 | 165 | 163 |
| 230 | 230 | 229 | 227 | 226 | 225 | 224 | 223 | 141 | 62 | 73 | 63 | 28 | 13 | 7 | 38 | 33 | 41 | 52 | 119 | 123 | 150 | 174 | 172 | 166 | 170 | 168 |
| 233 | 232 | 166 | 107 | 200 | 231 | 187 | 182 | 115 | 91 | 65 | 47 | 17 | 5 | 5 | 37 | 13 | 7 | 52 | 89 | 63 | 56 | 87 | 52 | 49 | 63 | 167 |
| 231 | 224 | 73 | 21 | 39 | 124 | 42 | 38 | 77 | 35 | 17 | 20 | 12 | 28 | 28 | 30 | 6 | 10 | 27 | 66 | 43 | 33 | 41 | 34 | 36 | 42 | 56 |
| 85 | 92 | 51 | 36 | 47 | 60 | 42 | 39 | 36 | 12 | 5 | 13 | 20 | 20 | 19 | 14 | 7 | 32 | 43 | 43 | 50 | 53 | 52 | 51 | 39 | 46 | 40 |
| 74 | 83 | 92 | 67 | 38 | 48 | 51 | 81 | 47 | 38 | 44 | 50 | 27 | 29 | 35 | 28 | 35 | 29 | 34 | 44 | 40 | 38 | 44 | 33 | 36 | 29 | 24 |
| 124 | 123 | 110 | 114 | 61 | 66 | 56 | 113 | 70 | 18 | 102 | 113 | 25 | 60 | 74 | 14 | 63 | 107 | 130 | 118 | 64 | 141 | 137 | 143 | 136 | 127 | 133 |
| 137 | 138 | 105 | 107 | 84 | 96 | 100 | 127 | 107 | 42 | 125 | 124 | 112 | 75 | 80 | 21 | 55 | 79 | 122 | 124 | 65 | 116 | 120 | 136 | 133 | 113 | 114 |
| 134 | 135 | 112 | 112 | 98 | 89 | 85 | 93 | 79 | 57 | 105 | 113 | 114 | 94 | 100 | 64 | 90 | 98 | 118 | 133 | 125 | 121 | 116 | 136 | 122 | 113 | 114 |
| 126 | 128 | 114 | 112 | 110 | 102 | 98 | 99 | 79 | 66 | 92 | 87 | 80 | 78 | 85 | 84 | 100 | 103 | 116 | 129 | 122 | 118 | 115 | 130 | 119 | 111 | 112 |

Collecte de l'information fréquentielle



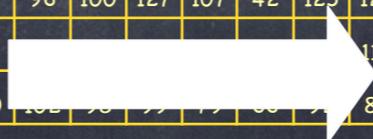
| | | | | | | | | | | | | | | | | | | | | | | | | | | |
|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|
| 175 | 175 | 176 | 175 | 175 | 171 | 166 | 164 | 163 | 161 | 158 | 157 | 155 | 154 | 150 | 147 | 143 | 141 | 139 | 136 | 134 | 131 | 128 | 125 | 122 | 120 | 118 |
| 178 | 178 | 178 | 178 | 177 | 174 | 173 | 168 | 167 | 164 | 162 | 160 | 157 | 156 | 153 | 151 | 147 | 145 | 143 | 140 | 138 | 135 | 131 | 127 | 125 | 125 | 122 |
| 180 | 180 | 180 | 180 | 180 | 178 | 176 | 173 | 171 | 167 | 167 | 164 | 162 | 160 | 156 | 155 | 151 | 149 | 146 | 143 | 140 | 139 | 136 | 132 | 130 | 128 | 127 |
| 185 | 184 | 184 | 184 | 182 | 180 | 179 | 177 | 175 | 171 | 170 | 167 | 165 | 163 | 160 | 160 | 155 | 153 | 151 | 148 | 145 | 142 | 139 | 137 | 136 | 133 | 129 |
| 194 | 194 | 191 | 187 | 185 | 185 | 183 | 182 | 180 | 176 | 175 | 174 | 170 | 170 | 153 | 163 | 161 | 158 | 155 | 152 | 150 | 147 | 144 | 141 | 139 | 136 | 136 |
| 199 | 199 | 197 | 194 | 192 | 188 | 185 | 184 | 183 | 180 | 178 | 176 | 173 | 171 | 145 | 165 | 163 | 160 | 157 | 155 | 152 | 149 | 147 | 144 | 141 | 139 | 137 |
| 202 | 201 | 203 | 199 | 199 | 195 | 192 | 188 | 187 | 184 | 183 | 180 | 177 | 176 | 116 | 171 | 167 | 166 | 161 | 159 | 158 | 155 | 153 | 149 | 146 | 143 | 141 |
| 206 | 205 | 205 | 204 | 201 | 200 | 198 | 197 | 183 | 144 | 169 | 184 | 177 | 178 | 81 | 174 | 172 | 168 | 160 | 164 | 162 | 159 | 156 | 153 | 151 | 149 | 145 |
| 209 | 210 | 211 | 209 | 207 | 205 | 204 | 205 | 151 | 72 | 97 | 123 | 89 | 156 | 72 | 173 | 169 | 157 | 140 | 172 | 169 | 167 | 163 | 160 | 157 | 154 | 151 |
| 215 | 215 | 213 | 213 | 213 | 209 | 209 | 208 | 147 | 65 | 86 | 98 | 66 | 76 | 79 | 169 | 165 | 122 | 85 | 154 | 174 | 172 | 169 | 166 | 163 | 160 | 158 |
| 225 | 225 | 223 | 220 | 219 | 217 | 215 | 214 | 127 | 50 | 60 | 88 | 50 | 27 | 27 | 85 | 108 | 83 | 57 | 127 | 178 | 178 | 175 | 171 | 168 | 165 | 163 |
| 230 | 230 | 229 | 227 | 226 | 225 | 224 | 223 | 141 | 62 | 73 | 63 | 28 | 13 | 7 | 38 | 33 | 41 | 52 | 119 | 123 | 150 | 174 | 172 | 166 | 170 | 168 |
| 233 | 232 | 166 | 107 | 200 | 231 | 187 | 182 | 115 | 91 | 65 | 47 | 17 | 5 | 5 | 37 | 13 | 7 | 52 | 89 | 63 | 56 | 87 | 52 | 49 | 63 | 167 |
| 231 | 224 | 73 | 21 | 39 | 124 | 42 | 38 | 77 | 35 | 17 | 20 | 12 | 28 | 28 | 30 | 6 | 10 | 27 | 66 | 43 | 33 | 41 | 34 | 36 | 42 | 56 |
| 51 | 36 | 47 | 60 | 42 | 39 | 36 | 12 | 5 | 13 | 20 | 20 | 19 | 14 | 7 | 32 | 43 | 43 | 50 | 53 | 52 | 51 | 39 | 46 | 40 | | |
| 92 | 67 | 38 | 48 | 51 | 81 | 47 | 38 | 44 | 50 | 27 | 29 | 35 | 28 | 35 | 29 | 34 | 44 | 40 | 38 | 44 | 33 | 36 | 29 | 24 | | |
| 110 | 114 | 61 | 66 | 56 | 113 | 70 | 18 | 102 | 113 | 25 | 60 | 74 | 14 | 63 | 107 | 130 | 118 | 64 | 141 | 137 | 143 | 136 | 127 | 133 | | |
| 105 | 107 | 84 | 96 | 100 | 127 | 107 | 42 | 125 | 124 | 112 | 75 | 80 | 21 | 55 | 79 | 122 | 124 | 65 | 116 | 120 | 136 | 133 | 113 | 114 | | |
| 112 | 112 | 98 | 89 | 85 | 93 | 79 | 57 | 105 | 113 | 114 | 94 | 100 | 64 | 90 | 98 | 118 | 133 | 125 | 121 | 116 | 136 | 122 | 113 | 114 | | |
| 114 | 112 | 110 | 102 | 98 | 99 | 79 | 66 | 92 | 87 | 80 | 78 | 85 | 84 | 100 | 103 | 116 | 129 | 122 | 118 | 115 | 130 | 119 | 111 | 112 | | |

Collecte de l'information fréquentielle

| | | | | | | | | | | | | | | | | | | | | | | | | | | |
|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|
| 175 | 175 | 176 | 175 | 175 | 171 | 166 | 164 | 163 | 161 | 158 | 157 | 155 | 154 | 150 | 147 | 143 | 141 | 139 | 136 | 134 | 131 | 128 | 125 | 122 | 120 | 118 |
| 178 | 178 | 178 | 178 | 177 | 174 | 173 | 168 | 167 | 164 | 162 | 160 | 157 | 156 | 153 | 151 | 147 | 145 | 143 | 140 | 138 | 135 | 131 | 127 | 125 | 125 | 122 |
| 180 | 180 | 180 | 180 | 180 | 178 | 176 | 173 | 171 | 167 | 167 | 164 | 162 | 160 | 156 | 155 | 151 | 149 | 146 | 143 | 140 | 139 | 136 | 132 | 130 | 128 | 127 |
| 185 | 184 | 184 | 184 | 182 | 180 | 179 | 177 | 175 | 171 | 170 | 167 | 165 | 163 | 160 | 160 | 155 | 153 | 151 | 148 | 145 | 142 | 139 | 137 | 136 | 133 | 129 |
| 194 | 194 | 191 | 187 | 185 | 185 | 183 | 182 | 180 | 176 | 175 | 174 | 170 | 170 | 153 | 163 | 161 | 158 | 155 | 152 | 150 | 147 | 144 | 141 | 139 | 136 | 136 |
| 199 | 199 | 197 | 194 | 192 | 188 | 185 | 184 | 183 | 180 | 178 | 176 | 173 | 171 | 145 | 165 | 163 | 160 | 157 | 155 | 152 | 149 | 147 | 144 | 141 | 139 | 137 |
| 202 | 201 | 203 | 199 | 199 | 195 | 192 | 188 | 187 | 184 | 183 | 180 | 177 | 176 | 116 | 171 | 167 | 166 | 161 | 159 | 158 | 155 | 153 | 149 | 146 | 143 | 141 |
| 206 | 205 | 205 | 204 | 201 | 200 | 198 | 197 | 183 | 144 | 169 | 184 | 177 | 178 | 81 | 174 | 172 | 168 | 160 | 164 | 162 | 159 | 156 | 153 | 151 | 149 | 145 |
| 209 | 210 | 211 | 209 | 207 | 205 | 204 | 205 | 151 | 72 | 97 | 123 | 89 | 156 | 72 | 173 | 169 | 157 | 140 | 172 | 169 | 167 | 163 | 160 | 157 | 154 | 151 |
| 215 | 215 | 213 | 213 | 213 | 209 | 209 | 208 | 147 | 65 | 86 | 98 | 66 | 76 | 79 | 169 | 165 | 122 | 85 | 154 | 174 | 172 | 169 | 166 | 163 | 160 | 158 |
| 225 | 225 | 223 | 220 | 219 | 217 | 215 | 214 | 127 | 50 | 60 | 88 | 50 | 27 | 27 | 85 | 108 | 83 | 57 | 127 | 178 | 178 | 175 | 171 | 168 | 165 | 163 |
| 230 | 230 | 229 | 227 | 226 | 225 | 224 | 223 | 141 | 62 | 73 | 63 | 28 | 13 | 7 | 38 | 33 | 41 | 52 | 119 | 123 | 150 | 174 | 172 | 166 | 170 | 168 |
| 233 | 232 | 166 | 107 | 200 | 231 | 187 | 182 | 115 | 91 | 65 | 47 | 17 | 5 | 5 | 37 | 13 | 7 | 52 | 89 | 63 | 56 | 87 | 52 | 49 | 63 | 167 |
| 231 | 224 | 73 | 21 | 39 | 124 | 42 | 38 | 77 | 35 | 17 | 20 | | | | | | | | | | | | | | | |



| | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
|-----|-----|-----|----|-----|-----|-----|----|-----|-----|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|
| 51 | 36 | 47 | 60 | 42 | 39 | 36 | 12 | 5 | 13 | | | | | | | | | | | | | | | | | | | |
| 92 | 67 | 38 | 48 | 51 | 81 | 47 | 38 | 44 | 50 | | | | | | | | | | | | | | | | | | | |
| 110 | 114 | 61 | 66 | 56 | 113 | 70 | 18 | 102 | 113 | | | | | | | | | | | | | | | | | | | |
| 105 | 107 | 84 | 96 | 100 | 127 | 107 | 42 | 125 | 124 | | | | | | | | | | | | | | | | | | | |
| 112 | 112 | 98 | | | | | | | 113 | | | | | | | | | | | | | | | | | | | |
| 114 | 112 | 110 | | | | | | | 87 | | | | | | | | | | | | | | | | | | | |



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